19 · Land Surveys, Instruments, and Practitioners in the Renaissance

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Introduction: The Situation in 1450

In the period around 1450 there were neither models nor methods for the complete cartographic depiction of the landscape. Scholars who lived in southern German monasteries and at the University of Vienna during the first half of the fifteenth century were involved in intensive work on Ptolemy's *Geography,* especially the calculating and collecting of coordinates.[1] The scholars' diverse activities, methods, and instruments were highlighted in two astonishing Munich codices by Fridericus.[2] But one cannot produce a large-scale map using coordinates alone. Even though the maps of the *Geography* came to fascinate the humanists, they were disappointed when they could find neither their place of birth nor their surrounding areas on the maps of ancient Germany. The pleas of Albert Magnus and Roger Bacon in the thirteenth century for high-quality maps could not be fulfilled simply by calculating coordinates.[3] The areas between towns with more or less known correct coordinates also had to be filled in on maps (for a reference map, see fig. 19.1).

In order to understand the significance of this absence, we must begin with the larger social and scientific context. Since the eleventh century, mathematical, astronomical, and even geographical learning had been spreading through Europe with the creation of universities. The speed increased considerably after 1400: twenty-eight universities existed in 1400, and eleven more opened over the next hundred years. Popes and sovereigns granted privileges, although they did not participate directly in scientific life. The sons of citizens and the lower nobility supported university life. These were the social groups that essentially profited from the spreading of education, especially by obtaining higher social ranks. The basic university courses, which every scholar was obliged to take, included the *artes liberales,* particularly mathematics and astronomy. We can trace one aspect of the enormous increase of scientific education in the corresponding increase of scientific manuscripts from the late medieval period that are conserved in libraries.

Another aspect of the period that deserves study was the popularity of anonymous practical manuals with scientific and mathematical content that were first written for the use of merchants (arithmetic), for architects and the building industry (geometry), and for navigators (astronomy). The demand for these practical manuals came from various sources: rich citizens and rich churchmen in growing towns such as Florence, Cologne, London, Paris, and Brugge; merchants; and also sovereigns, who hoped to use them to secure seafaring. These groups of people wanted to embellish their towns with great church buildings and teach their sons the known calculation methods useful for the exchange of merchandise and the art of navigation.

Another impetus for the increase of such knowledge was strong in all social ranks: the desire to know the future by means of astrology. Horoscopes required a good knowledge of the star constellations and the exact time and geographical coordinates of the interested individual's conception. The latter could not be taken from a map, but had to be determined spontaneously from the actual place. Later a collection of geographical coordinates could be gathered from horoscopes, as Peter Apian

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Abbreviations used in this chapter include: IMSS for the Istituto e Museo di Storia della Scienza, Florence; *Copernicus* for Uwe Müller, ed., *450 Jahre Copernicus "De revolutionibus": Astronomische und mathematische Bücher aus Schweinfurter Bibliotheken* (1993; reprinted Schweinfurt: Stadtarchiv Schweinfurt, 1998); *Kursächsische Kartographie* for Fritz Bönisch et al., *Kursächsische Kartographie bis zum Dreißigjährigen Krieg* (Berlin: Deutscher Verlag der Wissenschaften, 1990–); *Philipp Apian* for Hans Wolff et al., *Philipp Apian und die Kartographie der Renaissance,* exhibition catalog (Weißenhorn: A. H. Konrad, 1989); and *Rechenbücher* for Rainer Gebhardt, ed., *Rechenbücher und mathematische Texte der frühen Neuzeit* (Annaburg-Buchholz: Adam-Ries-Bund, 1999).

[1] Dana Bennett Durand, *The Vienna-Klosterneuburg Map Corpus of the Fifteenth Century: A Study in the Transition from Medieval to Modern Science* (Leiden: E. J. Brill, 1952).

[2] Munich, Bayerische Staatsbibliothek, Clm 14583 and 14783. See Armin Gerl, "Fridericus Amman," in *Rechenbücher,* 1–12, esp. 1–2.

[3] Uta Lindgren, "Die Geographie als Naturwissenschaft? Wie Albertus Magnus ein Forschungsdesiderat begründete," in *Köln: Stadt und Bistum in Kirche und Reich des Mittelalters,* ed. Hanna Vollrath and Stefan Weinfurter (Cologne: Böhlau, 1993), 571–87. On Bacon, see David Woodward, "Roger Bacon's Terrestrial Coordinate System," *Annals of the Association of American Geographers* 80 (1990): 109–22, and idem with Herbert M. Howe, "Roger Bacon on Geography and Cartography," in *Roger Bacon and the Sciences: Commemorative Essays,* ed. Jeremiah Hackett (Leiden: Brill, 1997), 199–222.

FIG. 19.1. REFERENCE MAP OF EUROPE.

did early in the sixteenth century. The practical manuals of the later Middle Ages did not equal the significance of the works of Euclid, Boethius, or Ptolemy, but were taught both at the universities and in the town schools. Part of their content became very useful for cartography as the desire for better maps intensified over the course of the fifteenth century. Therefore, we can state that the scientific ground for the cartographic depiction of the landscape was well prepared by scholars. The sovereigns' interest arose only later in the sixteenth century.[4]

In 1550, Sebastian Münster wrote: "Everything you measure must be measured by triangles." Although this sounds like a student's mnemonic, the question remains: when and where was this basic rule formulated?[5] No sources are known. We might consider Vienna, where in 1462–64 Johannes Regiomontanus began a purely mathematical treatise on trigonometric functions as used in astronomy from late Graeco-Roman times.[6] Trigonometric functions are, however, only one basis of triangulation. Evidence shows that others, such as Euclid's *Elements,* had been available to the Latin-speaking West since about 1120.[7] On the other hand, the methods of the *agrimensores* (Roman land surveyors) would not have been the origin of Münster's dictum. The original function of the *agrimensores* was to define the layout and the limits of newly founded towns and military camps, as well as to distribute land to campaign veterans, without seeking to reduce the earth's surface, especially in mountainous areas, to the geometrical surface of the globe, which was the basis of cartography.

When Johannes Stöffler, from whom we have the first instructions concerning practical geometry for surveyors, began his studies in Ingolstadt in 1472 when the university was newly founded, many masters from Vienna had come to lecture there. Other students who subsequently became cartographers made their way to Vienna, even though Vienna's initial heyday of studies in mathematics and astronomy was long past and the second had not yet begun.[8] Perhaps the University of Vienna ought to enjoy the reputation as the birthplace of practical geometry with its trigonometrical component.

In his "Ludi rerum mathematicarum" (ca. 1445), Leon Battista Alberti describes several procedures of land surveying in much more detail than in his "Descriptio urbis Romae." After explaining various procedures of practical geometry, such as calculating the height of a tower or the width of a river, Alberti instructs the reader to make a circular instrument at least a *braccia* wide (60–70 cm), then to divide the edge of the circle into twelve equal parts and each of the twelve parts into four parts, yielding a total of forty-eight parts (called degrees) for the circle, and then to divide each degree into four minutes. Alberti suggests that the instrument be used as follows. The observer selects a flat, high place from which he can see many landmarks, such as campaniles and towers, and lays the instrument flat on the ground. He then measures the

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[4] See chapter 26 in this volume.

[5] Sebastian Münster, *Cosmographei; oder, Beschreibung aller Länder* (Basel: Apud Henrichum Petri, 1550; reprinted [Munich: Kolb], 1992), XXVIII.

[6] Johannes Regiomontanus, *De triangulis omnimodis libri quinque* (Nuremberg, 1533). Leo Bagrow, in "The Maps of Regiomontanus," *Imago Mundi* 4 (1947): 31–32, points out that Regiomontanus had planned to publish maps himself; see also Ernst Zinner, *Regiomontanus: His Life and Work,* trans. Ezra Brown (Amsterdam: North-Holland, 1990), 55–60, esp. 56; Uta Lindgren, "Regiomontans Wahl: Nürnberg als Standort angewandter respektive praktischer Mathematik im 15. und beginnenden 16. Jahrhundert," *Anzeiger des Germanischen Nationalmuseums* (2002): 49–56; and Ernst Glowatzki and Helmut Göttsche, *Die Tafeln des Regiomontanus: Ein Jahrhundertwerk* (Munich: Institut für Geschichte der Naturwissenschaften, 1990).

[7] Menso Folkerts, "The Importance of the Latin Middle Ages for the Development of Mathematics," in *Essays on Early Medieval Mathematics: The Latin Tradition* (Aldershot: Ashgate, 2003), item I, esp. p. 6.

[8] Kurt Vogel, "Das Donaugebiet, die Wiege mathematischer Studien in Deutschland," and idem, "Der Donauraum, die Wiege mathematischer Studien in Deutschland," both in *Kleinere Schriften zur Geschichte der Mathematik,* 2 vols., ed. Menso Folkerts (Stuttgart: F. Steiner Verlag Wiesbaden, 1988), 2:571–73 and 2:597–659, and Christa Binder, "Die erste Wiener Mathematische Schule (Johannes von Gmunden, Georg von Peuerbach)," in *Rechenmeister und Cossisten der*

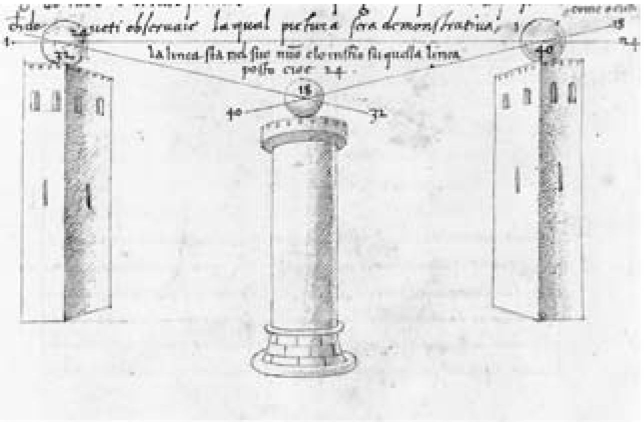

FIG. 19.2. ALBERTI'S METHOD FOR LAND SURVEYING, CA. 1455. From his "Ludi rerum mathematicarum." A circular disk with graduation of degrees and minutes served as a device with which to take bearings. Knowing the distance between stations, such as the two square towers of the castle at rear, and two angles, Alberti could calculate the distance from each tower to a third and perhaps inaccessible point, in this case a round tower.
Width of the original: 11.3 cm. By permission of Houghton Library, Harvard University (MS. Typ. 422/2).

bearing to each landmark in turn by standing two *braccia* (120–140 cm) behind the instrument, holding up a plumb line so it is coincident with both the center of the instrument and the landmark, and noting where the line intersects the instrument's scale of degrees and minutes. The observer then repeats this process at every other landmark in turn, including sighting back to the places he has already occupied. He will thus be able to fix the proportions of a series of triangles, such as that shown in figure 19.2. Alberti suggests that this method of backsighting is similar to the technique used by a navigator in guiding a ship along a particular wind ("sino a qui una nave avessea navicare"), but this allusion is not altogether clear. The known (measured) length of only one side is necessary to scale the whole triangle. Alberti then goes on to describe a method for fixing longer distances, for example, between cities such as Ferrara and Bologna. He clearly has an application for land surveying in mind, and its principles are those of triangulation.[9]

Completely independent of Alberti, Stöffler, and the activities at the University of Vienna, a number of *Rechenschulen* (schools of computation) were founded during the first third of the fifteenth century in southern Germany. Although they were primarily concerned with teaching commercial arithmetic, they also taught the basics of geometry, which was useful, for example, in the construction industry. Through these *Rechenschulen*, mathematical principles were widely distributed. Among other skills, these schools taught how to produce astronomical instruments; scholars no longer needed to manufacture instruments themselves, because craftsmen now specialized in instrumentmaking.

Land Surveys

ASTRONOMICAL METHODS

The summary of the various basic cartographic principles explained in the first book of Ptolemy's *Geography* had fundamental significance until the beginning of modern times.[10] Ptolemy had described the right astronomical methods, but did not use them to determine geographical longitudes. Instead he muddled through with the help of the route distances measured under the emperor Augustus. Although Muslim scholars did not create elaborate maps in the Ptolemaic tradition and using Ptolemaic definitions, their standard of exactness in astronomical observations and the means—that is to say, the instruments—they used first became known to European scholars in the tenth century and thus became models for Europe.

However, methods alone did not create reliable maps. European scholars faced a task that demanded much work, because Europe was densely populated and the landscape structures were of great variety. In addition to the objective difficulties of making maps on the basis of Ptolemy's methods, efforts were hampered by the fact that interest varied widely during the last four centuries of the Middle Ages and also from one country to another. The first effort to make measured maps—and we know little about how this was done—resulted in the thirteenth century in the portolan charts that were initially limited to the Mediterranean and Black Sea coasts. As a result of Ptolemy's *Geography,* astronomical observations took absolute priority over geographic observations. The geographic latitude of a location was calculated according to the height of the astronomical north pole. To calculate geographical longitude, it was necessary to carry out several observations of lunar eclipses at different places simultaneously, and all further geometrical observations were adapted to the fixed points so obtained. These geographical coordinates were entered into tables and added to globes and maps. All other observations were not of a geometrical nature.

During the fifteenth century, when the *Geography* was widely distributed, its explanation of methods gained importance, even though that information was somewhat

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frühen Neuzeit, ed. Rainer Gebhardt (Freiberg: Technische Universität Bergakademie Freiberg, 1996), 3–18.

[9] Leon Battista Alberti, "Ludi rerum mathematicarum," in *Opera volgari,* 3 vols., ed. Cecil Grayson (Bari: Gius. Laterza & Figli, 1960–73), 3:131–73 and 3:352–60. See also Pierre Souffrin, "La *Geometria pratica* dans les *Ludi rerum mathematicarum,*" *Albertiana* 1 (1998): 87–104.

[10] Ptolemy, *Geography,* 1.4.

sparse. In countries where scholars were active in the field of mathematics, such as Germany, Italy, France, England, Spain, and Portugal, tables of geographic coordinates were compiled and improved. The calculation of latitude through the height of the pole is mathematically sound. Peter Apian explained this method in *Cosmographicus liber,* which became extremely influential through its numerous reprintings.[11] Apian explained the adjustments needed to obtain latitude by observing the height of the midday sun,[12] a method that Ptolemy had acknowledged needed refinement.[13] Although Apian's contemporary, Oronce Fine, proposed a further method involving the introduction of the rising and setting of certain fixed stars, the sun-based methods remained extremely popular.[14] Sebastian Münster discussed both types of methods, but was not successful in hindering the continued use of the uncorrected sun height method. In succeeding centuries, the value of this method for measuring geographic latitude was often considerably less accurate than for longitude.[15]

Calculations of longitude based on lunar eclipses were very inexact due to the length of time of an eclipse and the impossibility of calculating exactly when it began and ended. Although some twelfth-century scholars in Latin-speaking countries knew the Islamic astronomical method of calculating the longitudinal difference of two locations by the simultaneous observation of the position of the moon in relation to that of a neighboring fixed star (the lunar distance method), one finds no mention of it in early writings on cosmology during the Renaissance.[16]

In the earliest editions of the *Cosmographicus liber* of 1524, Apian recommended only the lunar eclipse method for determining longitude,[17] but he introduced the lunar distance method in editions after 1540. Two observers must determine the difference in their local times before beginning the observations. The second observer can, however, be replaced by a lunar table (ephemerides), as explained in Apian's text. Oronce Fine covered only the lunar eclipse method in his *De cosmographia* of 1530,[18] but in the later, separate publication of *De mundi sphaera* he explained the method of comparing the meridial motion of the moon (when the moon passes the meridian of the observer) with the figure in the ephemerides for a central location.[19] On the other hand, Sebastian Münster described only the lunar eclipse method in his *Cosmography* of 1550, with the interesting variation that the observers should use clocks set to local time on the same evening.[20]

About the same time (1547), Reiner Gemma (Edelsteen), known generally as Reiner Gemma Frisius (i.e., of Friesland), suggested a new method for calculating longitude when on journeys, namely that of using a portable clock set to the local time of one's point of departure, which one would compare with the local time at one's destination.[21] He pointed out the limited value of this technique, because the mechanical clocks available then were so imprecise that they had to be corrected by comparing them to large water or sand clocks, which were able to run accurately for only a day. Since the eleventh century, local time had been using the astral clock or nocturnal.[22] Galileo Galilei's suggested method for longitude determination, using the changing eclipses of Jupiter's moons, had not been successful in practice.[23] It was also

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[11] Peter Apian, *Cosmographicus liber* (Landshut, 1524); citations here are from the 1540 Antwerp edition, *Petri Apiani Cosmographia,* chap. VIII, fol. X.

[12] Apian, *Cosmographia,* chap. IX, fol. XI.

[13] Ptolemy, *Almagest,* 3.4–9.

[14] Oronce Fine, *De cosmographia,* 1530 (published in 1532 as part three of his *Protomathesis*), fol. 146 v; see idem, *Orontii Finei Delphinatis, liberalivm disciplinarvm professoris regii, Protomathesis: Opus varium, ac scitu non minus utile quàm iucundum . . . ,* four parts: *De arimetica, De geometria, De cosmographia,* and *De solaribus horologiis* (Paris: Impensis Gerardi Morrhij et Ioannis Petri, 1532), and the unchanged Italian translation, *Opere di Orontio Fineo del Delfinato divise in cinque Parti: Arimetica, Geometrica, Cosmografia, e Oriuoli,* trans. Cosimo Bartoli (Venice, 1670).

[15] Boleslaw Szczesniak, "A Note on the Studies of Longitudes Made by M. Martini, A. Kircher, and J. N. Delisle from the Observations of Travellers to the Far East," *Imago Mundi* 15 (1960): 89–93, and Uta Lindgren, "Wissenschaftshistorische Bemerkungen zur Stellung von *Martinis Novus Atlas* Sinensis (1655)," in *Martino Martini S. J. (1614–1661) und die Chinamission im 17. Jahrhundert,* ed. Roman Malek and Arnold Zingerle (Sankt Augustin: Institut Monumenta Serica, 2000), 127–45, esp. 130.

[16] Juan Vernet Ginés, "El nocturlabio," in *Instrumentos astronómicos en la España medieval: Su influencia en Europa* (Santa Cruz de la Palma: Ministerio de Cultura, 1985), 52–53, and Ernst Zinner, *Deutsche und niederländische astronomische Instrumente des 11.–18. Jahrhunderts* (1956; Munich: H. C. Beck, 1979), 164. See also Gerald R. Tibbetts, "The Beginnings of a Cartographic Tradition," and David A. King and Richard P. Lorch, "Qibla Charts, Qibla Maps, and Related Instruments," both in *HC* 2.1:90–107, esp. 103 n. 67, and 189–205. Fundamental research was done and published by J. L. Berggren in *Episodes in the Mathematics of Medieval Islam* (New York: Springer, 1986), and by E. S. Kennedy and H. M. Kennedy in *Geographical Coordinates of Localities from Islamic Sources* (Frankfurt am Main: Institut für Geschichte der Arabisch-Islamischen Wissenschaften an der Johann Wolfgang Goethe-Universität, 1987).

[17] Apian, *Cosmographia,* fol. XVI.

[18] Fine, *De cosmographia,* fol. 145v.

[19] Oronce Fine, *Orontij Finei Delphinatis, . . . De mundi sphaera, sive, Cosmographia . . . ,* first separately published in Paris in 1542; I used a later edition (Paris: Apud Michaelem Vascosanum, 1555), fol. 48v–49v.

[20] Münster, *Cosmographei,* XXXIII.

[21] Reiner Gemma Frisius, *De principiis astronomiæ et cosmographiæ, deq[ue] vsu globi ab eodem editi: Item de orbis diuisione, & insulis, rebusq[ue] nuper inuentis* (1530; Paris, 1547), citations from the Antwerp edition (1584), 239. There is disagreement on the proper form of the author's name; I use Reiner Gemma Frisius.

[22] Vernet Ginés, "El nocturlabio," and Zinner, *Deutsche und niederländische astronomische Instrumente,* 164.

[23] Emil Bachmann, *Wer hat Himmel und Erde gemessen?: Von Erdmessungen, Landkarten, Polschwankungen, Schollenbewegungen, Forschungsreisen und Satelliten* (Thun: Ott, 1965), 86.

TABLE 19.1 Differences between Longitude and Latitude Values from Four Coordinate Tables and Modern Values[a]

Places of the Same Longitude According to Münster, 1550	Modern Values		Oronce Fine, 1541		Johannes Stöffler, 1518		Peter Apian, 1524/1540		Ptolemy's *Geography* (Ulm, 1482)	
	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.
Basel	7°36′	47°33′	29°45′	47°45′	0h 8m	48°	24°22′	47°41′	28°	47½°
Strasbourg	7°35′	48°35′	30°15′	48°45′			24°30′	48°45′	27½°	48¾°
Kaiserslautern	7°47′	49°27′					24°44′	49°22′		
Koblenz	7°36′	50°21′					23°56′	50°25′		
Münster/Westfalen	7°37′	51°58′	32°00′	52°05′	0h 6m	51°	24°08′	52°00′		
Groningen	6°35′	53°13′	29°50′	53°15′	0h 10m	53°	22°54′	53°16′		

[a] Sebastian Münster, in both the Latin and the German editions of his *Cosmography* of 1550, held that the six towns listed were at the same longitude, and modern values confirm that they are relatively close with the exception of Groningen. However, the longitude values for these towns differed considerably within the coordinate tables of Fine, Stöffler, Apian, and Ptolemy. The departure point was the Canary Islands for Fine, Apian, and Ptolemy. Stöffler's departure point was Tübingen (where one minute of time equates to fifteen minutes of arc). Modern values are from Greenwich.

an illusion that the Earth's magnetism could be used to measure geographical longitude. Nevertheless, as more research in the field of geomagnetism was carried out during the second half of the sixteenth century, the hope that it could provide a satisfactory method intensified.[24] This idea was still being preached by Athanasius Kircher in the middle of the seventeenth century.[25]

Peter Apian listed more than fifty towns in Bavaria with their longitude and latitude in an expanded coordinate table.[26] Comparison with other tables, in which individual regions were not so fully documented, shows how widely different the measured results could be (table 19.1). Astronomically measured fixed points were the basis for drawing modern maps. The geographic features lying between these points were cartographically fixed using a variety of different methods. These methods drew more and more from geometrical principles.

TERRESTRIAL METHODS: LAND SURVEYORS, GEOMETERS, CARTOGRAPHERS

Theoretical Works

The Hellenistic-astronomical method of mapmaking had, at roughly the same time, a terrestrial counterpart among the Romans in the techniques of the *agrimensores* (field surveyors), although the two methods were never combined before the end of the fifteenth century. The special feature of the techniques of the *agrimensores* was that they could be used to calculate areas. Geometrically speaking, this method relied on visualizing all areas as combinations of easily constructed squares and rectangles. Triangular areas could not be calculated. The written works of these *agrimensores,* which also discussed other topics, were known during the Middle Ages and were copied and distributed. In the fourteenth century, the lawyer Bartolo da Sassoferrato used the knowledge of the *agrimensores* after a disastrous flooding of the Tiber in order to regulate the rights of land possession in the newly formed river valley.[27] The most important work on land surveying, "De limitibus constituendis" by Caius Julius Hyginus (ca. A.D. 100), was copied eleven times during the sixteenth century.[28] *The Maner of Measurynge*

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[24] Hans Gunther Klemm, *"Von der Krafft und Tugent des Magneten": Magnetismus-Beobachtungen bei den humanistischen Mathematikern Georg Hartmann und Georg Joachim Rheticus* (Erlangen: Hans Gunther Klemm, 1994), and Giovanni Battista Della Porta, *Magiæ natvralis libri viginti* (Frankfurt: Apud Andreæ Wecheli heredes, Claudium Marnium & Ionn. Aubrium, 1591), citations from the German translation, *Magia naturalis; oder, Haus- Kunst- und Wunder-Buch,* 2 vols., ed. Christian Knorr von Rosenroth (Nuremberg, 1680), bk. VII, chap. XXXVIII, 961–63. This method soon found application in England; E. G. R. Taylor mentioned it with regard to Thomas Digges (ca. 1579), William Borough (1581), and Robert Norman (1581) in *The Mathematical Practitioners of Tudor & Stuart England* (1954; reprinted London: For the Institute of Navigation at the Cambridge University Press, 1967), 324–25.

[25] Athanasius Kircher, *Magnes siue de arte magnetica opvs tripartitvm* (Rome: Ex typographia Ludouici Grignani, 1641), 504–6: "Modus faciendi Mappa[m] Geographico-Magneticam" (Way of making geographic-magnetic maps). Earlier Kircher had explained how one could find geographic longitudes at sea from his declination tables.

[26] Apian, *Cosmographia,* fol. XXXIII r/v.

[27] Bartolo da Sassoferrato, "Tractus Tyberiadis o de fluminibus," 1355, and Fritz Hellwig, "Tyberiade und Augenschein: Zur forensischen Kartographie im 16. Jahrhundert," in *Europarecht, Energierecht, Wirtschaftsrecht: Festschrift für Bodo Börner zum 70. Geburtstag,* ed. Jürgen F. Baur, Peter-Christian Müller-Graff, and Manfred Zuleeg (Cologne: Carl Heymanns, 1992), 805–34, esp. 805–7.

[28] Menso Folkerts and Hubert Busard, *Repertorium der mathematischen Handschriften* (forthcoming).

All Maner of Land, by Richard Benese, appeared in London in 1537, and this again discussed the methods of the *agrimensores.*[29]

None of these methods, however, found its way into mapmaking. Indeed, surveyors were content to rely on practical geometry, based on Euclid's *Elements,* and to use the principles of triangles therein for triangulation purposes. Euclid was not mentioned in the context of land measurement before the end of the fifteenth century, and it is still not known who was responsible for this shift. Sebastian Münster's instruction to measure using triangles is found in the middle of this process. Münster, a professor of Hebrew studies, cannot be considered the originator, but he was an effective propagator.

Nobody expressed the idea that triangulation was the decisive method of choice as clearly as Münster had. He could well have learned this from his tutor in Tübingen, Johannes Stöffler, although not all the techniques that he described can be found in Stöffler's treatise on practical geometry, *De geometricis mensurationibus rerum,* first printed in Oppenheim in 1513 by Jakob Köbel.[30] The individual examples used in the following period varied enormously in these tracts, but this says little about the principles employed. Perhaps the wide range of examples had more to do with the commercial success sought for the booklets. In his work Stöffler explained, using a number of examples, how inaccessible distances could be calculated. One side of a triangle must be measured using a measuring stick (*pertica*), and angles must be observed. Most of the examples given were based on the similarity of triangles, on proportions or relations, and on the use of the rule of three (*Regeldetri*). Stöffler also explained a number of examples using a variation of Jacob's staff where the longstaff and crosspiece are roughly divided, and he said that the position of the surveyor must be carefully chosen to fit into the calculations. The last examples were based on the use of shadow squares with "umbra versa" (vertical shadow) and "umbra recta" (horizontal shadow). These examples assumed knowledge of the cotangent function, even if they did not use the angle values. This had the advantage of avoiding a possible source of error that arose when people measured angles with the simple devices of the time.

In 1522 Jakob Köbel published a German version of Stöffler's geometry.[31] The examples using the shadow square method were not in the first edition; in the second edition, which appeared posthumously in 1536, they were included.[32] Köbel's work popularized the adapted Jacob's staff and explained that the mirror functioned as a sort of bearing device.

This early modern triangulation got its name from the use of triangles in the surveying of land. It is comprised of a combination of various geometrical components: (1) teachings based on the triangle methods found in Euclid's *Elements;* (2) the use of trigonometric functions, with which the sides of a right-angled triangle can be ascertained when one side and one angle are known; (3) practical rules deduced from one or both sources.

Peter Apian employed triangles covering surprisingly large areas.[33] He compiled a table from which it was possible to extract the length of a degree of latitude as one traveled away from the equator.[34] In his first example of how to calculate the distance between Erfurt in Thüringia and Santiago de Compostela in Galicia, he recommended the use of a globe. From this he was able to read off the coordinates. Apian's other examples worked along the same lines: the distance between two locations was computed using the known coordinates. However, Apian used plane trigonometry and left the spherical shape out of his equation, calculating the distance between Jerusalem and Nuremberg using a sine table. In each case he dealt with a systematically calculated example, not with an explanation of the method.

Oronce Fine was already aware of Stöffler's methods of measuring the location of inaccessible points explained in Stöffler's *De geometricis mensurationibus rerum* (1513).[35] In Fine's *De geometria* of 1530, he explained a series of examples using smaller distances in which one side of a triangle could not be measured directly and had to be calculated, for example, the height of a tower visible on the other side of a stretch of water or the depth of a well.[36] Geometrically, he used proportions of suitably chosen triangles. In many cases he used the methods of Euclid and to some extent trigonometric functions. At least one distance had to be measured in each case, sometimes up to three distances.[37] In some examples, he con-

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[29] Sarah Tyacke and John Huddy, *Christopher Saxton and Tudor Map-Making* (London: British Library Reference Division, 1980), 18; Richard Benese, *This Boke Sheweth the Maner of Measurynge of All Maner of Lande, as well of Woodlande, as of Lande in the Felde, and Comptynge the True Nombre of Acres of the Same: Newlye Inuented and Compyled by Syr Rycharde Benese* (Southwark: James Nicolson, 1537); Valentine Leigh, *The Moste Profitable and Commendable Science, of Surueying of Landes, Tenementes, and Hereditamentes* (1577; reprinted Amsterdam: Theatrum Orbis Terrarum, 1971); and Taylor, *Mathematical Practitioners,* 168 and 312.

[30] Stöffler's *De geometricis mensurationibus rerum* was bound together with his *Elvcidatio fabricæ vsvsqve astrolabii* (Oppenheim: Jacobum Köbel, 1513).

[31] Josef Benzing, *Jakob Köbel zu Oppenheim, 1494–1533: Bibliographie seiner Drucke und Schriften* (Wiesbaden: Guido Pressler, 1962), 60 and 70.

[32] Jakob Köbel, *Geometrei, vonn künstlichem Messen vnnd Absehen allerhand Hoͦhe . . .* (Frankfurt, 1536). The last of this booklet's twelve editions appeared in 1616; citations are from the 1608 edition, *Geometrey, von kůnstlichem Feldmessen vnnd Absehen allerhandt Hoͦhe. . .* (Frankfurt: S. Latomo, 1608).

[33] Apian, *Cosmographia,* fol. XVIIIv–XXII.

[34] Apian, *Cosmographia,* fol. XVIIIv.

[35] Eberhard Knobloch, "Oronce Finé: Protomathesis," in *Copernicus,* 188–90.

[36] Fine, *De geometria* (part 2 of his *Protomathesis*), fol. 49v–76v.

[37] Fine, *De geometria,* fol. 72.

structed a large geometrical quadrat with sides about one meter in length as a reference length. Elsewhere he used his own eye level as a reference. The values of the angles did not play a role in these examples. In addition to the geometrical square, Fine employed a geometrical quadrant, i.e., a quadrant with an inscribed square and plumb bob, a Jacob's staff based on Stöffler's *baculus geometricus,* and a suitably positioned mirror.[38] Fine left open the combination in which these measuring and calculating procedures should be carried out. Later, in his *De mundi sphaera,* he devoted a chapter entirely to the construction of maps, with the example of a map of the French coast of the Mediterranean Sea measuring about ten by ten centimeters.[39]

Gemma published a work in 1533 on land surveying methods that after 1540 was printed with subsequent editions of Apian's *Cosmographicus liber.*[40] The aim of the work was to explain how to construct a map of a particular area with the aid of land measurements. It contained information on astronomical as well as terrestrial principles and the necessary instruments.[41] Many editions of the *Cosmographicus liber* were published, with translations in Spanish, French, and Flemish. There were also anonymous editions as well as editions that other authors had attributed to themselves. The work became by far the most widespread manual for mapmakers and instrumentmakers in the sixteenth and seventeenth centuries.

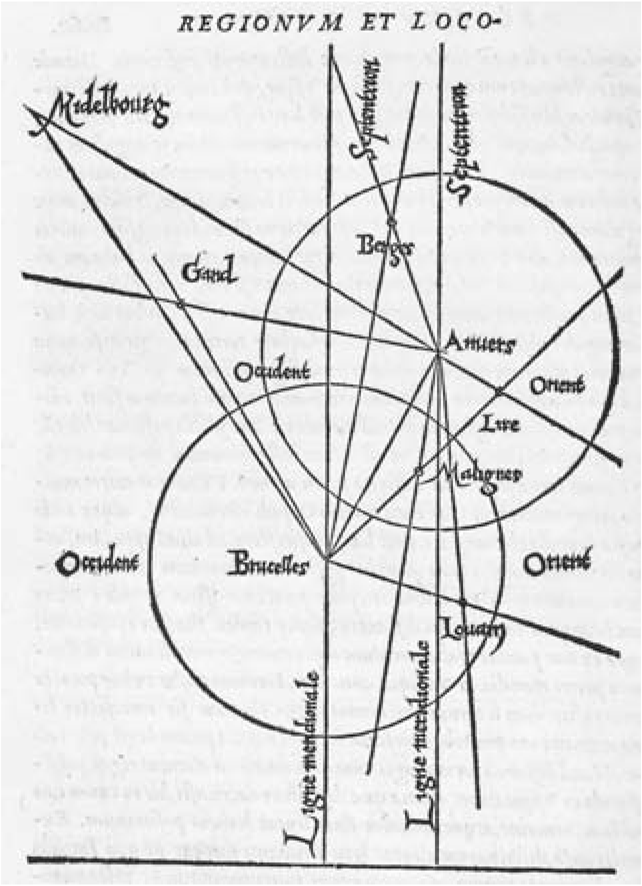

FIG. 19.3. TRIANGULATION OF THE BRUSSELS AND ANTWERP ENVIRONS. Gemma took the bearings of the surrounding towns from towers in Brussels and Antwerp and drew the directions on circular disks of paper that he put together later. He then found the location of the distant sites by extending the lines until they intersected.
Size of the original: 15.5 × 11 cm. From Reiner Gemma Frisius, *Libellvs de locorum* . . . (Paris, 1553), 60v. Photograph courtesy of the Universiteitsbibliotheek Leiden (20077, A16).

Gemma's most important example relates to the determination of the positions of the towns near Brussels and Antwerp through the use of angle measurements made from particular viewpoints (fig. 19.3).[42] To measure the angles, Gemma used a compass, a circle divided into quarters (each further divided into ninety degrees), an alidade, and a circular sheet of paper on which to record the observations for each city.[43] On top of each observation tower, he first used the compass to define the meridian and properly orient the circle; he then used the circle and the alidade to sight to each distant town and then drew each bearing onto one sheet of paper whose center represented the tower. At home, he placed the circular sheets onto a larger piece of paper, oriented them properly, and extended the lines of bearings until they intersected, thereby defining the location of each town. He could readily alter the map's scale by moving the two circular sheets closer together or farther apart; the actual scale could be determined from just one distance measured between one of the observation points and one of the landmarks. In other examples, in which the distance between two places was already known, all that was needed was to draw circles using a pair of compasses, the points of intersection marking the locations of the two places. The observations were to be continually repeated until the whole of a province or neighborhood was surveyed. In using this technique, Gemma was only one step away from inventing the surveyor's plane table, which he would have developed if he had used an instrument and (replaceable) sheets together.

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[38] Fine, *De geometria,* fol. 72.

[39] Fine, *De mundi sphaera,* bk. 5, chap. 6, fol. 53v–54v: "De constructione chartarum chorographicarum." In *De Cosmographia* (1530), mapmaking had been treated even more extensively and illustrated by a rough sketch of the border of France (bk. 5, chap. 7, fols. 154–55).

[40] Reiner Gemma Frisius, *Libellvs de locorum* . . . (1533; Antwerp, 1540). The number of editions of the *Cosmographia* issued can no longer be established, although it was at least sixty, partly because many of the translations bear only the name of the new publisher, not those of the authors. We do know that reprints were published well into the seventeenth century. I have used the 1540 edition.

[41] Uta Lindgren, "Johannes de Sacrobosco: Sphera volgare novamente tradatto," in *Copernicus,* 221–22.

[42] Gemma Frisius, *Libellvs de locorum,* fol. XLVIIIv. For a fuller discussion, see pp. 1297–98 in this volume.

[43] Gemma Frisius, *Libellvs de locorum,* fol. XLVIIv: "Index cum perspicillis aut pinnulis."

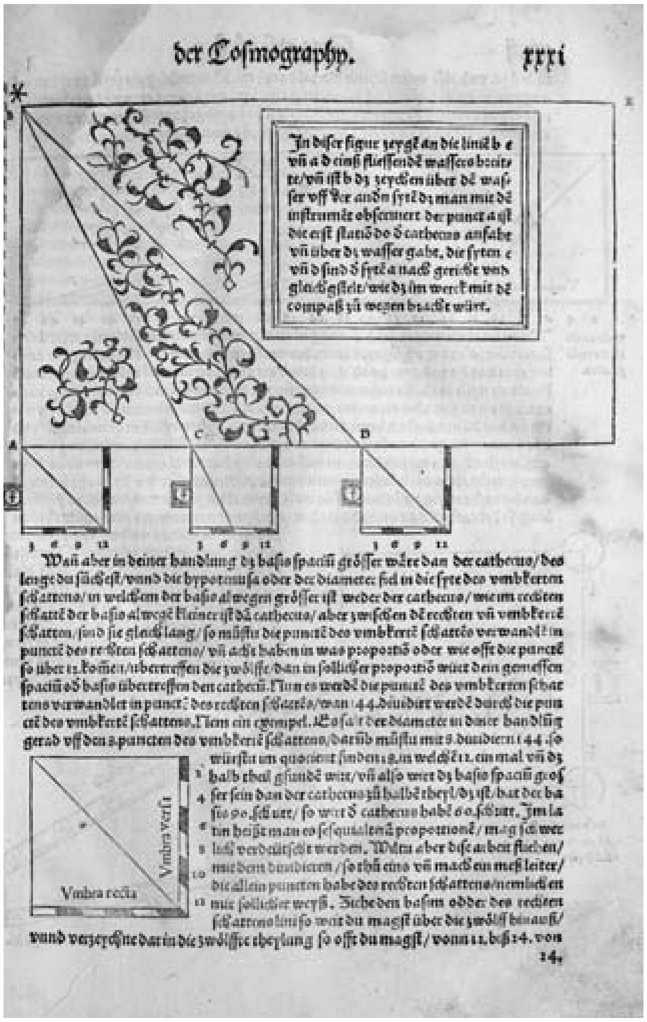

FIG. 19.4. GEOMETRICAL QUADRAT AND HEURISTIC MODEL, 1550. The shadow square (lower left) graphically represents trigonometrical functions; the "umbra recta" stands for the cotangent. To determine the unknown width of the Rhine (AB upper left), Münster determined points C and D along the river bank such that the umbra recta was either six units (half of its full length) or twelve units (equal to its full length), respectively, so that AB was either twice AC or equal to AD.
Size of the original: ca. 31.2 × 19.2 cm. Sebastian Münster, *Cosmographei; oder, Beschreibung aller Länder . . .* (Basel: Apud Henrichum Petri, 1550), 31. Photograph courtesy of the Special Collections Research Center, University of Chicago Library.

Gemma prided himself on the precision of his method, whereby no error could be noticed for distances of up to one hundred German miles (ca. 750 km).[44] When calculating larger distances or greater areas, the problem of establishing the meridian by using a magnetic compass needle could have an effect. The most important factor for accuracy was the measurement of the base line, for which Peter Apian gave instructions.[45]

Sebastian Münster, whose *Cosmography* had been available in German since 1544, explained three ways in which unknown lengths of the sides of a triangle could be ascertained. These can be found in his explanation of the basic terminology of cartography, based on the work of Ptolemy.[46] In the first two examples, the two angles and their connecting base line are measured. These values—at reduced scale—are transferred to paper. The third corner of the triangle is formed by the intersection of the sides. The required distance between this corner point and the base point of the observer can now be taken from the drawing. This graphic method of problem solving was not unusual in other areas during the Middle Ages, but it was rare in land measurement.[47] Although in principle correct, it suffered under Münster from various practical inaccuracies. In the first example, the distances from Offenburg to Basel and Thann were much too great to allow exact bearings to be taken. Münster used an instrument for measuring angles similar to that used by Gemma except that the magnetic compass needle was incorporated. In his second example, the instrument was a *triquetum* (a three-armed instrument also called a *Dreistab*) with two angle measurement devices and a compass. The magnetic compass was used, as by Gemma, to determine the line of meridian, from which the other angles were calculated. This second example worked with proportions.

For his third example, Münster used a geometrical square (the shadow square) that had two sides labeled "umbra recta" and "umbra versa," just as Stöffler had described in his work, to determine a length common to two triangles without actually measuring any angles (fig. 19.4). In Münster's example, the unknown length was the width of the river Rhine near Basel; moving along the bank of the river, and taking repeated bearings with the alidade of the quadrat against the umbra recta, he identified those points where the value of the umbra recta was six and

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[44] Gemma Frisius, *Libellvs de locorum,* fol. LIIII.

[45] Apian, *Cosmographia,* fol. XVII, and also Köbel, *Geometrey.* Uta Lindgren, "Astronomische und geodätische Instrumente zur Zeit Peter und Philipp Apians," in *Philipp Apian,* 43–65, esp. 50.

[46] Münster, *Cosmographei,* XXI. The 1544 German edition was much shorter than the 1550 edition. For Münster's work on land measurement problems, see Uta Lindgren, "Kosmographie, Landkarten und Vermessungslehre bei Sebastian Münster," in *Sebastian Münster (1488–1552): Universalgelehrter und Weinfachmann aus Ingelheim,* ed. Gabriele Mendelssohn, exhibition catalog ([Ingelheim]: Historischer Verein Ingelheim, 2002), 27–39.

[47] Matthias Schramm, "Ansätze zu einer darstellenden Geometrie bei Schickhard," in *Wissenschaftsgeschichte um Wilhelm Schickard,* ed. Friedrich Seck (Tübingen: J. C. B. Mohr [Paul Siebeck], 1981), 21–50, esp. 21–25. In the sixteenth century, the word "geodesy" meant land surveying or the measuring of land, not that branch of applied mathematics that determines the figures and areas of large portions of the earth's surface and the figure of the earth as a whole. See John Dee, "Mathematicall Praeface," in *The Elements of Geometrie of the Most Auncient Philosopher Evclide of Megara,* by Euclid, trans. Henry

twelve units (out of twelve units), such that the distance he had moved along the bank was either half of or equal to the width of the river. Because he did not actually measure any angles or use tables to determine a cotangent for use in a calculation, Münster very cleverly eliminated possible sources of error.

In 1574 Erasmus Reinhold, a doctor and astronomer from Saalfeld and son of the mathematician and astronomer from Wittenberg of the same name, published his *Bericht vom Feldmessen und vom Markscheiden.*[48] Besides presenting an introduction to basic calculation with a table of squares for the numbers from one to four thousand for computing their square roots, he also explained commonly recurring examples of land surveying that relied on the similarity of triangles and that could be solved using the rule of three (*Regeldetri*) and with the calculation of relationships or proportions. This book, like other teaching books, harked back to the field of Euclidian geometry. Reinhold also introduced his readers to the possibilities of using triangulation with the table of sine values supplied by him to solve surveying problems. Here the focus was on the calculation of the sides of a triangle and its area when given measured angles. Reinhold's required instruments were a measuring stick, a rope for measuring the distances, and a "Compast" angle-measuring instrument—a large circle equipped with a magnetic compass and an alidade. With these, angles could be measured within ten minutes.

All the aforementioned authors incorporated triangulation into the process of map production. Willebrord Snellius had a different objective when he attempted to measure the length of one degree of the great circle along a meridian in his *Eratosthenes Batavus.*[49] His base points were Bergen op Zoom and Alkmaar. Somewhere between these, near Leiden, he measured a short base line. Working from this base line, he proceeded to set out a network of triangles on both sides, of which he measured all three angles in order to calculate the lengths of the other two sides. The instrument he used was a large quadrant. The method of calculation (using the sine function to determine the lengths of the sides) was not described in his report, but he was convinced of the need for exactness.[50] He found this undertaking very tiring and justified his perseverance by saying that the result was "for the public good."[51] Unfortunately, the measurement was much too short at only 107 instead of 111 kilometers for one degree. Whether the triangulation network approach described by Snellius was more influential for land surveys than the writings of Apian and Gemma has been variously debated.

A few years before his death, Wilhelm Schickard wrote instructions in his *Kurtze Anweisung* for making maps and explained to simple travelers how they could report information that would keep him and others from taking long and difficult journeys.[52] The work is a de facto mixture of learned teaching, personal experience, and explanation. As models Schickard mentioned the techniques of Sebastian Münster, Aegidius Tschudi, David Seltzlin, Wolfgang Lazius, Georg Sandner, Sebastian von Rotenhan, Johannes Mellinger, and Bartholomäus Scultetus, all of whom had "made maps according to geometrical principles."[53] Actually this was true only for Scultetus, and Schickard omitted Philipp Apian.[54] Schickard incorrectly alleged that Gemma's method of using a disk divided into 360 degrees was too imprecise.[55] Instead, he recommended using a simpler device, whereby the circle on a dial would be repeatedly divided until there were ninety-six sections, and then a pointer with an alidade and a magnetic needle compass would be added.[56] He then applied the method suggested by Gemma (in his Antwerp-Brussels example).

Schickard described a third method based on tables of known distances in and around Tübingen. Using a pair of compasses, he drew circles that intersected at the relevant locations. It was not a precise technique, because the distances in Schickard's tables were known only in terms of hours. The calculation of coordinates, which Schickard only partially explained, should be used only as a second choice, when the triangulation measuring points are too far apart.[57]

While Snellius did not consider the construction of maps, and consequently his *Eratosthenes Batavus* does

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Billingsley (London: Printed by Iohn Daye, 1570), a.iij verso: "Of these Feats . . . is Sprong the Feate of *Geodesie,* or Land Measuring." However, because its modern meaning follows the second definition, the word is avoided here.

[48] Herbert Wunderlich, *Kursächsische Feldmeßkunst, artilleristische Richtverfahren und Ballistik im 16. und 17. Jahrhundert: Beiträge zur Geschichte der praktischen Mathematik, der Physik und des Artilleriewesens in der Renaissance unter Zugrundelegung von Instrumenten, Karten, Hand- und Druckschriften des Staatlichen Mathematisch-Physikalischen Salons Dresden* (Berlin: Deutscher Verlag der Wissenschaften, 1977), 24–32, and Erasmus Reinhold, *Bericht vom Feldmessen und vom Markscheiden* (Erfurt, 1574).

[49] Willebrord Snellius, *Eratosthenes Batavus: De terræ ambitus vera quantitate* (Leiden, 1617).

[50] Snellius, *Eratosthenes Batavus,* 169–70.

[51] Snellius, *Eratosthenes Batavus,* 171.

[52] Wilhelm Schickard, *Kurtze Anweisung wie künstliche Landtafeln auß rechtem Grund zu machen und die biß her begangne Irrthumb zu verbessern, sampt etlich new erfundenen Voertheln, die Polus Hoehin auffs leichtest und doch scharpff gnug zu forschen* (Tübingen, 1669), 9.

[53] Schickard, *Kurtze Anweisung,* 1.

[54] On Bartholomäus Scultetus, see Werner Stams, "Bartholomäus Scultetus—Kartenmacher und Bügermeister in Görlitz," *Mitteilungen/Freundeskreis für Cartographica in der Stiftung Preussischer Kulturbesitz e.V.* 14 (2000): 26–35.

[55] Schickard, *Kurtze Anweisung,* 14.

[56] Schickard, *Kurtze Anweisung,* 16.

[57] Schickard, *Kurtze Anweisung,* 18–21.

not supply any relevant instructions, Daniel Schwenter explicitly considered surveying in the field, both for civil and military architecture and also for cartography, in his *Ohne einig künstlich geometrisch Instrument,* issued as the second part of Schwenter's *Geometria practica nova* (1617; 2d ed. 1623), and *Mensula Praetoriana,* issued as the third part of Schwenter's *Geometria practica nova* (1626). Schwenter's works contain a greater number of examples of how an inaccessible stretch of land can be calculated using triangulation.

In addition to all the examples mentioned in his *Mensula Praetoriana,* there is also a complete example of measuring the area of a piece of land.[58] In order to do this, Schwenter climbed numerous towers, established the meridian using a magnetic compass, and then for every location placed a new sheet of paper on his measuring table and entered the bearing of each important feature in the area. This is the same technique that Gemma taught in his examples using observation points in Brussels and Antwerp, but here it is in expanded form. The joining together of all the sheets was done at home, exactly as Gemma had done it, onto one large sheet, whereby the bearings were extended as far as their points of intersection.

The various works just detailed are characterized by their relative independence from one another and their originality, even if they ultimately employ the same triangulation methods. Many works were based on them.[59] Comparing the known maps of that time, we must acknowledge a gap between the theoretical knowledge and education of cartographers and surveyors, on the one hand, and cartographic practice, on the other. This did not mean that there was little demand for maps. On the contrary, Renaissance sovereigns grew more interested in cartographic representations of their realms. For example, the case of Philipp Apian is well documented.[60] Duke Christoph of Württemberg, the son of Duke Ulrich, who had called Stöffler to the University of Tübingen, proudly showed a map of his country to his cousin, Duke Albrecht V of Bavaria, when the latter visited him in 1554. Albrecht had been sent to Peter Apian for a scientific education in the company of the latter's son, Philipp Apian. The experience provoked a great interest in maps in Albrecht, who sent Philipp Apian to Christoph to inspect the map of Württemberg and determine whether he could create a similar map of Bavaria. Deceived by what he had seen, Philipp Apain returned to report that he could easily surpass the Württemberg map—in a "cosmographic manner," because the Württemberg map was only a painting. As a result, he received the famous commission for his map of Bavaria, which was finished in 1563 with a hand-drawn map measuring 5 × 5 meters and followed in 1568 by a woodcut printing measuring 1.7 × 1.7 meters.

The interest of Renaissance rulers in obtaining good maps was various. The visualization of the sovereign's heritage often gave him a better knowledge of its range. Knowledge promoted better fiscal control and budget planning. It also served a military purpose, especially allowing realistic planning for specific distances and landscapes. For judicial matters, better knowledge helped clarify possession rights and allowed the sovereign to declare the favored grounds for the most cherished privilege of the sovereign: hunting.

Another factor that contributed to the gap between cartographic theory and execution had to do with the demands of the landscape itself. The outdoor job of a surveyor was hard and dangerous. Philipp Apian was only in his twenties when he performed the measuring work for his map throughout Bavaria, and he chose to do it only in the summer months, while he taught at the University of Ingolstadt in the winter. His brother, Timotheus, who assisted him, died from a riding accident shortly before the end of the work.[61] Gerhardus Mercator was already in his fifties when in 1563 he undertook measurements in

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[58] Daniel Schwenter, *Mensula Praetoriana: Beschreibung deß nutzlichen geometrischen Tischleins, von dem Mathematico M Johanne Praetorio S. erfunden* (Nuremberg, 1626), 84–90.

[59] Marco Mauro published a work in 1537 that at first glance seems to be an Italian translation of Johannes de Sacrobosco's "De sphaera" (first half of the thirteenth century) but the enlargement of which reveals that it was heavily influenced by Apian's *Cosmographicus liber* with appendixes by Gemma; see Marco Mauro, *Sphera volgare novamente tradotto* (Venice: Zanetti, 1537), and Lindgren, "Johannes de Sacrobosco," 221. Cosimo Bartoli follows Oronce Fine, but also refers to the works of Albrecht Dürer, Gemma Frisius, Philipp Apian, Johannes Stöffler, and Georg von Peuerbach; see Cosimo Bartoli, *Del modo di misvrare le distantie, le superficie, i corpi, le piante, le prouincie, le prospettiue, & tutte le altre cose terrene, che possono occorrere a gli huomini, secondo le uere regole d'Euclide, & de gli altri piu lodati scrittori* (Venice: Francesco Franceschi Sanese, 1589), with earlier editions published in Venice in 1559 and 1564. See also *La corte il mare i mercanti: La rinascita della Scienza. Editoria e società. Astrologia, magia e alchimia* ([Milan]: Electa Editrice, 1980), and Eberhard Knobloch, "Praktische Geometrie," in *Maß, Zahl und Gewicht: Mathematik als Schlüssel zu Weltverständnis und Weltbeherrschung,* ed. Menso Folkerts, Eberhard Knobloch, and Karin Reich, exhibition catalog (Weinheim: VCH, Acta Humaniora, 1989), 123–85, esp. 130–31. Giovanni Pomodoro, in *Geometria prattica* (Rome: Giovanni Martinelli, 1603), follows Euclid (Knobloch, "Praktische Geometrie," 144–45), as do William Bourne (Tyacke and Huddy, *Christopher Saxton,* 23, and Taylor, *Mathematical Practitioners,* 176) and Leonard Digges (Leonard Digges, *A Geometrical Practise Named Pantometria* [London, 1571]; R. A. Skelton, *Saxton's Survey of England and Wales: With a Facsimile of Saxton's Wall-map of 1583* [Amsterdam: Nico Israel, 1974], 24 n. 38; and Taylor, *Mathematical Practitioners,* 166–67). Although from the title it appears that Paul Pfinzing's *Methodus Geometrica* (1589) belongs to this group, his method is not mathematical, and the instruments are emphasized in his work. The explanations Pfinzing gives are remarkably fragmentary.

[60] Gertrud Stetter, "Philipp Apian 1531–1589: Zur Biographie," in *Philipp Apian,* 66–73, esp. 70, and 205.

[61] Stetter, "Apian," 71.

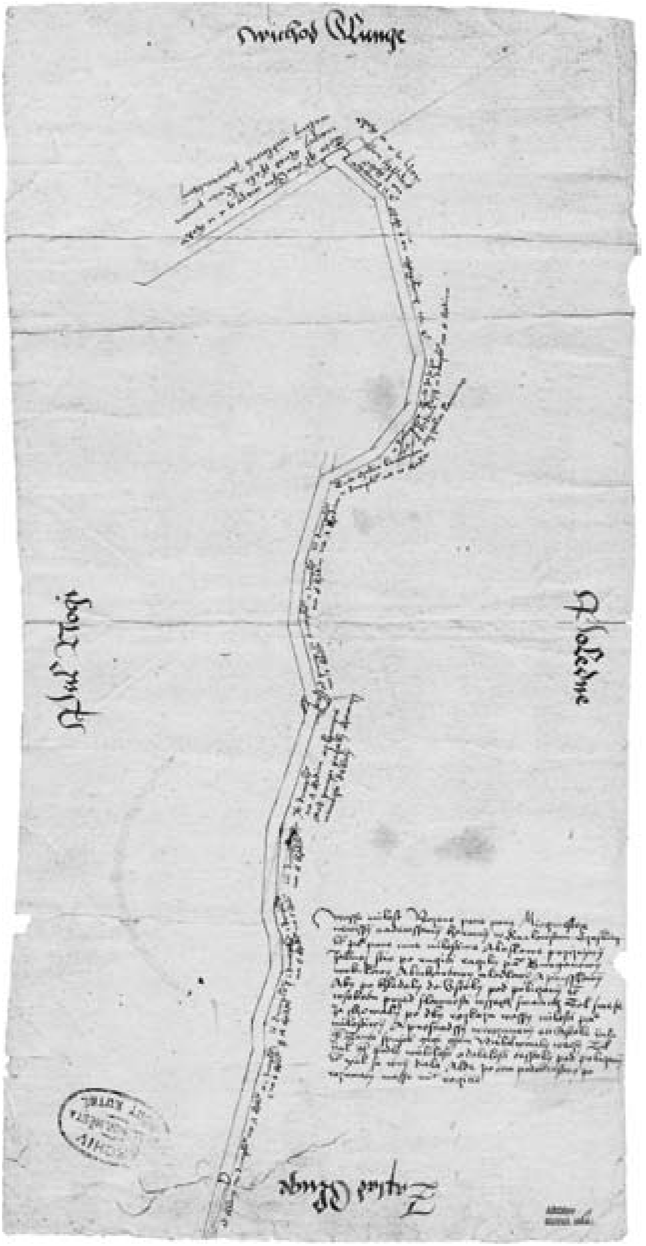

FIG. 19.5. OLDEST SURVIVING MINING DRAFT OF BOHEMIA NEAR KUTNÁ HORA (KUTTENBERG), DRAWN BY ZIKMUND PRÁŠEK, 1534. This schematic drawing depicts only one dimension of the gallery and is supplemented by verbal additions.

Size of the original: 41 × 20.5 cm. Photograph courtesy of the State Regional Archives in Prague-State District Archive in Kutná Hora (Kuttenberg) (Collection of Documents of Mining and Mint Offices, Nr. 147).

Lotharingen for his map. After finishing the map, he fell seriously ill, recovering slowly. He never again returned to outdoor jobs, but sent his sons and grandsons instead. Snellius complained bitterly about the hard conditions of outdoor work, and neither Münster nor Schickard ever thought of doing it by themselves. Münster sent out letters to beg for information, and Schickard worked on his booklet as an instruction for others who would do the outdoor job. In the eighteenth century, Peter Anich died from exhaustion after doing such work. The combination of factors—the increasing demand for maps by Renaissance sovereigns and the physical challenges of land measuring work—begin to explain the ways in which cartographic theory was complicated by the practice on the ground.

Mine Surveying Methods

Land measurement and staking ownership in the mining industry began above ground in 1300. The oldest mining rights, e.g., those of Kutná Hora (Kuttenberg), clearly laid down the legal significance of the mine surveying process. Map paintings, drawings, or sketches of mines, however, have been preserved only since the sixteenth century. From 1529 there is a panorama-style sketch with mining boundaries from Fichtelberg Mountain in the Erzgebirge (Saxony),[62] and from 1534 a pit ground plan from the vicinity of Kutná Hora (fig. 19.5).[63] All representations from the sixteenth and early seventeenth centuries are exceedingly stylized. Unlike the case of land survey maps, in which distances can be verified on the landscape, modern verification is not possible in the case of mine surveys, because the mines are no longer accessible. The same standards cannot be applied to mine surveying carried out in the early modern era as were applied to land surveying. The methods employed remind one rather of the legal maps that appeared about the same time (discussed later).

In Schwenter's *Ohne einig künstlich geometrisch Instrument,* geometrical techniques are included for two mine surveying methods, which combine measurements on the slope and underground.[64] Again, Schwenter started with similar triangles and referred only to Euclid. The two examples were designed not for the production of maps, but as aids to making decisions in connection with digging pits or tunnels (fig. 19.6).

Georg Agricola studied theology and later medicine, and in 1527 became the town physician of Jáchymov (St. Joachimsthal), the center of Saxony's silver mining district. From 1533 Agricola was a four-time mayor of Chemnitz. He published several books on mineralogy and geology. Agricola's chief work, *Vom Bergwerck,* on Saxony's mining and geology, has an illustration of an

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[62] Hans Brichzin, "Augenschein-, Bild- und Streitkarten," in *Kursächsische Kartographie,* 1:112–206, esp. 137.

[63] Jan Urban, "Alte böhmische Bergbaukarten," *Der Anschnitt* 22, no. 4 (1970): 3–8, esp. 4.

[64] Daniel Schwenter, *Ohne einig künstlich geometrisch Instrument allein mit der Meßrute und etlichen Stäben das Land zu messen,* 2d ed. (Nuremberg, 1623), 76–80.

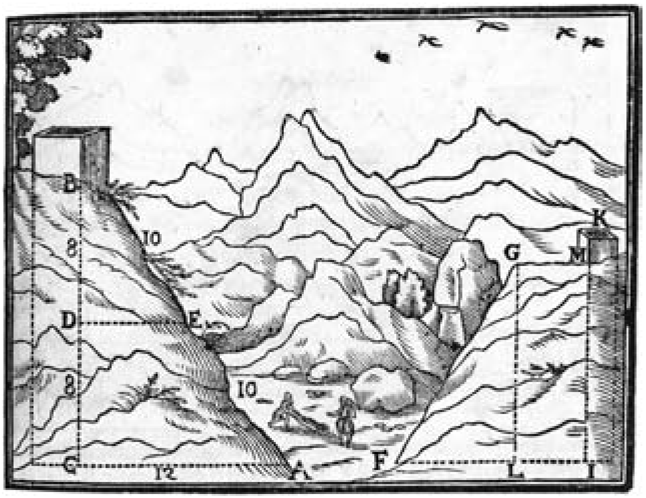

FIG. 19.6. METHOD FOR MINING SURVEY, 1617. The schematic figure shows the design of the vertical pit and the mountain's slope.
Size of the original: ca. 8.6 × 11.2 cm. Daniel Schwenter, *Ohne einig künstlich geometrisch Instrument allein mit der Meßrute und etlichen Stäben das Land zu messen,* which is the second part of his *Geometria practica nova* (Nuremberg, 1617), 65. Photograph courtesy of the BL (717.f.7.[4.]).

iron angle bracket and a quadrant mounted inside a circular frame with a pendulum pointer (fig. 19.7).[65] It is depicted in the landscape in such a way that one surmises it must have been used on the slope. The text makes no mention of this; it mentions only a magnetic compass, string, and writing utensils for use underground. Agricola does not explain any of the methods used.

Erasmus Reinhold's 1574 *Bericht vom Feldmessen und vom Markscheiden* has several examples using two separate systems: first, using similar triangles according to Euclid, and second, using sine tables or umbra tables.[66] He therefore suggested the same methods used in land surveying. Besides the measuring rods and a water level, he employed instruments such as a quadrant with an integrated compass, a sighting tube (instead of the alidade), and a "pit level," i.e., a semicircle divided into degrees and fitted with a plumb line. Due to the limited lighting available underground, how far triangulation could be carried out in practice is questionable. This is also the case for the use of the "mathematical measuring box," a mathematical precision instrument constructed by Tobias Volckmer (1589) of the Dresden Kunstkammer, for which a set of operating instructions was written in 1591.[67] This included instructions for use in the mine surveying business.[68]

Town and City Surveys

Geometrical techniques had been used since the sixteenth century for town and city surveys. Augustin Hirschvogel

FIG. 19.7. MINE SURVEYING INSTRUMENT, 1557. A large quadrant with plumb bob is mounted into a circular frame to permit measurements of slope.
Size of the original: ca. 22.6 × 14.3 cm. Georg Agricola, *Vom Bergwerck* (Basel, 1557), fol. CV. Photograph courtesy of the BL (443.h.6[2]).

employed the methods of Gemma for his 1547 plan of Vienna,[69] and it is suspected that Johann van der Corput

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[65] Georg Agricola, *Vom Bergkwerck,* commentary by Hans Prescher (Basel, 1557; reprinted Weinheim: Acta Humaniora der VCH, 1985), CV.

[66] Wunderlich, *Kursächsische Feldmeßkunst,* 25–32.

[67] Wunderlich, *Kursächsische Feldmeßkunst,* 104–8.

[68] Wunderlich, *Kursächsische Feldmeßkunst,* 114.

[69] Karl Fischer, "Augustin Hirschvogels Stadtplan von Wien, 1547/1549, und seine 'Quadranten,'" *Cartographica Helvetica* 20 (1999): 3–12, and idem, "Stadtpläne und Veduten Wiens im 16. Jahrhundert," in *8. Kartographiehistorisches Colloquium Bern 3.–5. Oktober 1996: Vorträge und Berichte,* ed. Wolfgang Scharfe (Murten: Cartographica Helvetica, 2000), 185–90.

used the same for Duisburg.[70] How exactly Jakob Sandtner created his extremely accurate town models in relief remains unclear, but their quality strongly suggests the use of triangulation. He made five such models of Bavarian towns between 1568 and 1574. Sandtner insisted on being given written authorization to make observations (from high places) in the fortified city of Ingolstadt.[71]

INSTRUMENTATION EMPLOYED

The instruments preserved in the Kunstkammern collections in places such as Dresden, Florence, and Cassel astound the modern observer with their versatility and the preciseness of their manufacture.[72] To take these instruments as the starting point from which to reconstruct their function would pose considerable difficulty. The analysis becomes somewhat easier and nearer to the surveyors' reality if we begin with the tracts on surveying and the instruments illustrated therein. However, in principle, no method of land surveying relied on one particular instrument.

The instruments used in land surveying were employed in carrying out three tasks: measuring time, measuring distances, and measuring angles, including determining the bearings of a position. Other aids were the magnetic compass, the plane table, the ephemerides,[73] and the surveyors' assistants.

MEASURING TIME

In order to determine longitudinal coordinates, it was necessary to establish the local time of the observer. To do this, Islamic scholars developed an instrument that employed the daily orbit of Ursa Minor and Major around the north pole. This astral clock, also known as the nocturnal, had been used since the end of the eleventh century in the Christian West.[74] This is one of the instruments that can be found illustrated on the title page of Peter Apian's book of instruments (1533). Apian also depicted and described it in his *Cosmographicus liber* (fig. 19.8; and fig. 19.9 for another example).[75]

In 1547, Gemma pointed out the fundamental significance of portable mechanical clocks, i.e., pocket watches, for determining longitude.[76] In order to be of any use, they had to be continually checked due to their inefficiency, so their implementation in practice was more or less illusory. The importance of precision can be easily seen when one considers that the heavens appear to move at a speed of fifteen degrees per hour, i.e., one degree in four minutes, and one minute in four seconds. More reliable, but unfortunately not always portable, were water and sand clocks. In order for the correct time to be read from a sundial, an attempt was made to take into account on the dial itself the seasonal differences in the movement

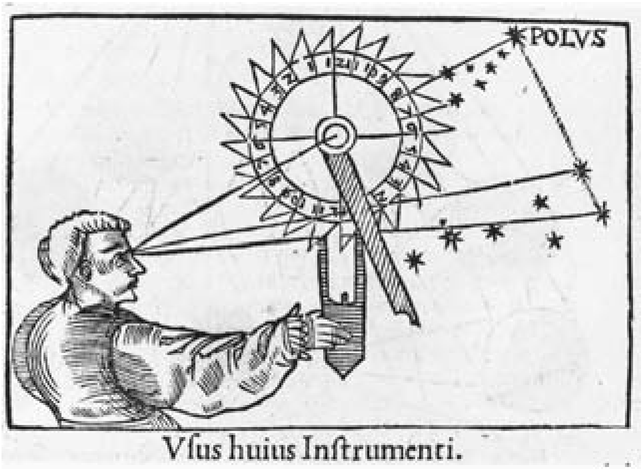

FIG. 19.8. ASTRAL CLOCK (NOCTURNAL) IN APIAN'S *COSMOGRAPHIA*, 1540. The night sky can be used as a clock because of the way in which the stars apparently rotate about the Pole Star in Ursa Minor in a constant manner. The pointers in Ursa Major—the two stars that consistently lie on a straight line with the Pole Star—are the hand for this celestial clock. The nocturnal recreates and calibrates the celestial clock: the observer sights the Pole Star through a hole in the center of the circle, moves the arm to cover the pointers, and then reads off the local time from the position of the arm on the scale. From Peter Apian, *Petri Apiani Cosmographia*, ed. Reiner Gemma Frisius (Antwerp, 1540), fol. XLVIv. Photograph courtesy of the BL (531.g.10[2]).

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[70] Günter von Roden, *Duisburg im Jahre 1566: Der Stadtplan des Johannes Corputius* (Duisburg-Ruhrort: Werner Renckhoff, 1964), and Joseph Milz, "Der Duisburger Stadtplan von 1566 des Johannes Corputius und seine Vermessungsgrundlagen," *Cartographica Helvetica* 11 (1995): 2–10.

[71] Alexander Freiherr von Reitzenstein, *Die alte bairische Stadt in den Modellen des Drechslermeisters Jakob Sandtner, gefertigt in den Jahren 1568–1574 im Auftrag Herzog Albrechts V. von Bayern* (Munich: Georg D. W. Callwey, 1967).

[72] Anthony Turner, *Early Scientific Instruments: Europe 1400–1800* (London: Sotheby's Publications, 1987); Gerard L'Estrange Turner, *Elizabethan Instrument Makers: The Origins of the London Trade in Precision Instrument Making* (Oxford: Oxford University Press, 2000); Mara Miniati, ed., *Museo di storia della scienza: Catalogo* (Florence: Giunti, 1991); J. A. Bennett, *The Divided Circle: A History of Instruments for Astronomy, Navigation and Surveying* (Oxford: Phaidon, Christie's, 1987); A. W. Richeson, *English Land Measuring to 1800: Instruments and Practices* (Cambridge: Society for the History of Technology and M.I.T. Press, 1966); and Edmond R. Kiely, *Surveying Instruments: Their History* (1947; reprinted Columbus, Ohio: Carben Surveying Reprints, 1979).

[73] Johannes Stöffler had continued Regiomontanus's ephemerides up to 1556. Reinhold calculated the ephemerides anew and published them in 1551, and Kepler did as well in 1624 during the period discussed; see Johannes Stöffler, *Ephemeridum reliquiae Ioannis Stoeffleri Germani, superadditis novis usque ad annum Christi 1556. durantibus Petri Pitati Veronensi Mathematici . . .* (Tübingen, 1548); Erasmus Reinhold, *Prutenicae tabulae coelestium motuum* (Tübingen, 1551); and Johannes Kepler, *Tabulæ Rudolphinæ* (Ulm, 1627).

[74] Vernet Ginés, "El nocturlabio."

[75] Apian, *Cosmographia.*

[76] Gemma Frisius, *De principiis astronomiæ,* chap. XIX, 239.

FIG. 19.9. ENGLISH NOCTURNAL, CA. 1600. The modifications to this instrument minimize the observational errors that originate in the daily and annual movements of Ursa Minor around the Pole Star, which does not precisely mark the north pole.
Size of the original: 7.7 cm diameter. Photograph by Franca Principe, courtesy of the IMSS (inv. no. 2500).

FIG. 19.10. SUN QUADRANT, SEVENTEENTH CENTURY. Astronomers tried to design dial plates or sun quadrants so as to take into account the steady change in the sun's arc through the year. An additional problem was finding the exact north-south direction to orient the instrument correctly; this required a rod or gnomon to be stuck in the hole in the upper part of the quadrant.
Size of the original: maximum radius 5.5 cm. Photograph by Franca Principe, courtesy of the IMSS (inv. no. 3251).

of the sun (figs. 19.10 and 19.11).[77] Assuming that the movement of the sun had been sufficiently researched, the dial still presented a difficult task for the mathematician.[78] As a consequence of these problems, precise measuring of time using the sundial seldom occurred in a surveying context.

MEASURING DISTANCES

In the simplest cases, Jakob Köbel and Peter Apian began by explaining how to measure distance by paces,[79] and Sebastian Münster suggested that the time taken to cover his base from Basel to Thann be converted into miles. No particularly exact results could be thus attained for the purposes of land measurement. Ropes, used for measuring distances of several meters, were not a good measuring instrument due to their sag. Chains were not much better, because slack caused by their great weight could not be completely avoided even when several strong assistants were at work.[80] Much more accurate results could be obtained by using measuring poles, although in those days they were made of wood, which changed slightly in length depending on the weather.[81] This was a minor cause of error. For short distances, one used a measuring stick or ruler, sometimes made of metal.[82] Ease of use and relatively exact measurements over a distance of several miles were possible with odometers. These mechanical measuring devices on wagons go back to late antiquity and the engineer and architect Vitruvius Pollio.[83] August I of Saxony received an odometer of fire-gilded brass, a precisely manufactured device mounted on a traveling coach, from Christoph Trechsler the Elder in 1584.[84] In order to measure a useful stretch for triangulation purposes, the carriage could not drive along any street but had to travel a long, straight stretch or across large (presumably previously harvested) fields. The possibilities for such a precise device to serve surveying purposes were therefore seriously limited. It could obviously be put to other uses, however, for example, to measure the distance between two or more towns.

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[77] Apian applied the form of a poplar wood leaf to his geometrical design: Peter Apian, *Instrument Buch* (1533; reprinted Leipzig: ZA-Reprint, 1990), and Lindgren, "Astronomische und geodätische Instrumente," 44–45.

[78] For Bartolomeus Scultetus, see Uta Lindgren, "Bartholomaeus Scultetus: Gnomonice De Solariis," in *Copernicus,* 265–66, and Zinner, *Deutsche und niederländische astronomische Instrumente,* 532. For Johannes Hommel, see Uta Lindgren, "Johannes Hommel: Gnomonik (1561)," in *Copernicus,* 348, and Zinner, *Deutsche und niederländische astronomische Instrumente,* 388. For Andreas Schöner, see Karin Reich, "Andreas Schöner: Gnomonice," in *Copernicus,* 264–65, and Zinner, *Deutsche und niederländische astronomische Instrumente,* 527–28. And for Johannes Schöner, see Zinner, *Deutsche und niederländische astronomische Instrumente,* 528–29.

[79] Lindgren, "Astronomische und geodätische Instrumente," 50.

[80] Lindgren, "Astronomische und geodätische Instrumente," 53.

[81] Lindgren, "Astronomische und geodätische Instrumente," 52.

[82] Wunderlich, *Kursächsische Feldmeßkunst,* 21–22.

[83] Preserved in the Mathematics-Physics Salon, Dresden. Lindgren, "Astronomische und geodätische Instrumente," 54.

[84] Wunderlich, *Kursächsische Feldmeßkunst,* 60–63.

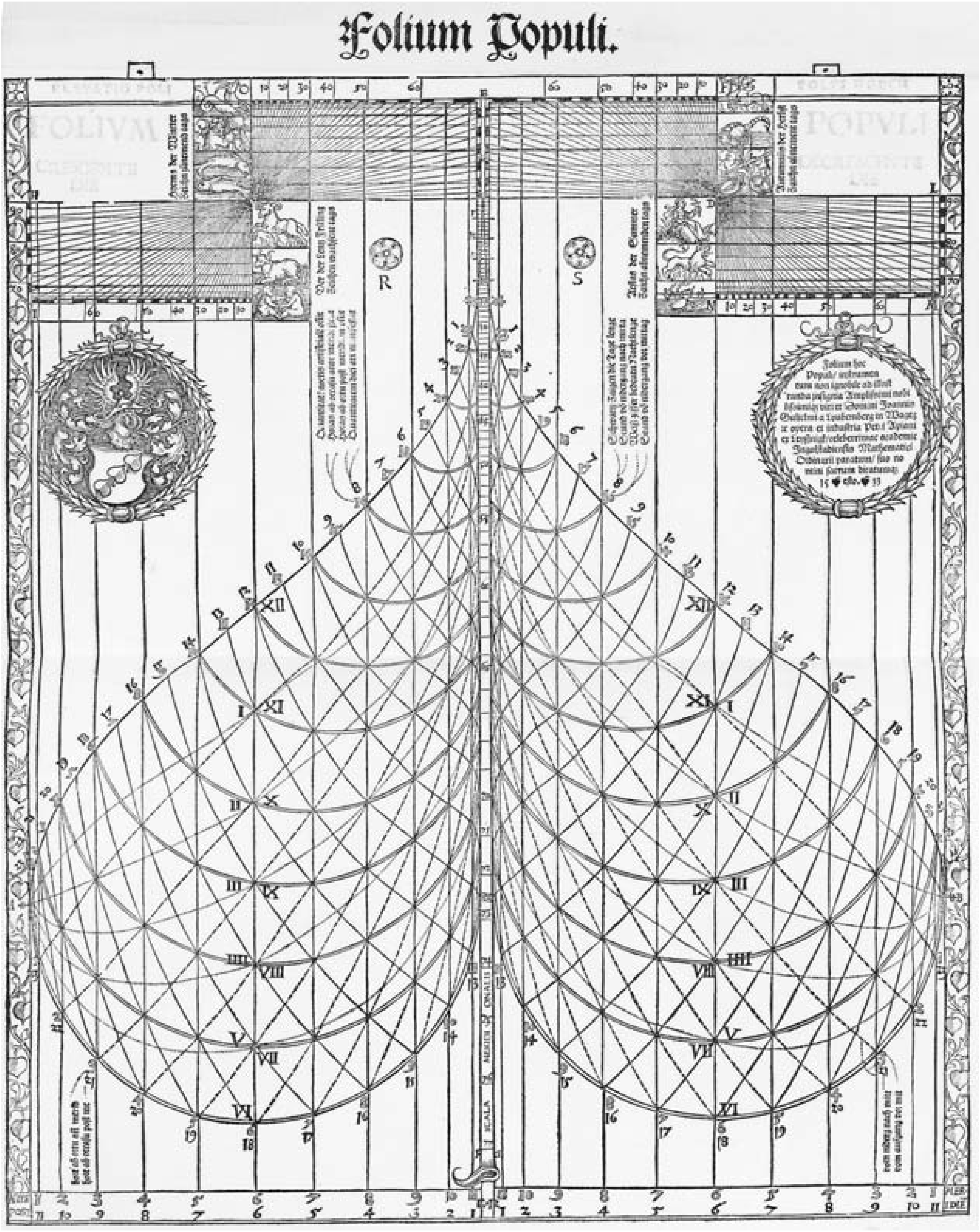

FIG. 19.11. SUNDIAL IN THE FORM OF A POPLAR LEAF, 1533. The use of this sundial, illustrated by Apian, required an adjustable, three-part gnomon to take into account latitude and month.

Size of the original: 31 × 24.5 cm. Peter Apian, *Instrument Buch* (Ingolstadt, 1533). Photograph courtesy of the Beinecke Rare Book and Manuscript Library, Yale University, New Haven (Shelfmark QB85 A63+).

MEASURING ANGLES

Traditional Instruments

Traditionally, angle measurement was significant only in the study of astronomy. This changed during the fifteenth century. Its use in land surveying has been described in teaching manuals since the sixteenth century.

When Peter Apian published his *Instrument Buch* in 1533, the most important instruments, namely the quadrant, Jacob's staff (or cross staff), and geometrical quadrat, were as useful for astronomy and navigation as for land surveying. That dual use was also practiced by the successful authors and instrumentmakers who resided in Nuremberg, e.g., Georg Hartmann and Johannes Schöner.[85] But early land measurement, like navigation, did not need as exact and complicated instruments as astronomy. Until the middle of the century, the land surveyors, with their own special methods, preferred to make their own instruments. In some cases, weight was taken into consideration. Consequently, surveyors who did not want to tax themselves during their fieldwork preferred light instruments, i.e., instruments with staffs made of wood.

The quadrant is a quarter of a circle fitted with an alidade on one of the arms and with a plumb bob that hangs from the center point of the circle.[86] Even Ptolemy described a quadrant.[87] In the Middle Ages this was preferred to the astrolabe for certain areas of work because its graduated scale was larger in proportion to a whole circle instrument. The very large wall quadrants of Ulugh Beg in Samarkand[88] and Tycho Brahe in Uraniborg[89] are well known. In order to calculate the distance of the moon from one of the fixed stars, one needed only the scale of degrees. In land measurement, the quadrant was especially appropriate for the determination of heights, as the plumb showed the angle,[90] while one could better observe the horizontal angle using the quadrant fitted with a pointer.

The Jacob's staff was designed to measure angles between the horizon and a star. The idea can probably be traced back to Hipparchus (190–120 B.C.), but it was improved in the Middle Ages by Levi ben Gerson (1288–1344).[91] Very clear instructions for its production and use can be found in Peter Apian's *Cosmographicus liber.*[92] With these, anyone could make and use his own. However, the manufacture of the finely divided scale required a considerable amount of skill. This instrument had no other function than that of angle measurement.[93] Due to its simple construction and ease of use, the Jacob's staff was widely used. Even simpler was Fine's *baculus geometricus,* with its coarse scale.

The geometrical quadrat had been used since the eleventh century in the measurement of angles.[94] One can see it being employed in this function on the title page of Peter Apian's *Instrument Buch*. In describing it, Apian made special mention of its use in the calculation of distances.[95] As he used it, it was simply a square frame, while other authors had a square wooden board. At least two sides were divided into as many units as possible; Apian divided his into one thousand units. In one corner a movable arm, the *regula,* was fixed. This strip and at least one of the sides was fitted with an alidade. These devices were produced with a side length of up to approximately one meter.

Determining the Bearings of a Position

In addition to calculating angles and measuring distances, this geometrical square was also used in combination with a magnetic compass for taking bearings of observations in land surveying. Using the methods applied by Gemma and the plane table, it was then no longer necessary to calculate any angles. Finally, a special function of the geometrical quadrat, the shadow square, could also be used for measuring methods based on the umbra recta (cotangent) function. This is explained clearly by Sebastian Münster in his *Cosmography* (fig. 19.4). Very often the theoretical approach cannot be easily recognized, because it was usual to calculate the examples without explaining the mathematical principles behind them.[96]

The Brunswick (Braunschweig) instrumentmaker Tobias Volckmer manufactured a *quadratum geometricum*

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[85] Hans Gunther Klemm, *Georg Hartmann aus Eggolsheim (1489–1564): Leben und Werk eines fränkischen Mathematikers und Ingenieurs* (Forchheim: Ehrenbürg-Gymnasium, [1990]), and idem, *Der fränkische Mathematicus Johann Schöner (1477–1547) und seine Kirchehrenbacher Briefe an den Nürnberger Patrizier Willibald Pirckheimer* (Forchheim: Ehrenbürg-Gymnasium, 1992).

[86] Zinner, *Deutsche und niederländische astronomische Instrumente,* 203–7.

[87] Ptolemy, *Almagest,* 1.12.

[88] Stephen Finney Mason, *Geschichte der Naturwissenschaft in der Entwicklung ihrer Denkweisen,* trans. Bernhard Sticker (1953; reprinted Stuttgart: Alfred Kröner, 1961), 125.

[89] J. R. Christianson, *On Tycho's Island: Tycho Brahe and His Assistants, 1570–1601* (Cambridge: Cambridge University Press, 2000), 118–19.

[90] Apian, *Instrument Buch,* Ciii verso–Civ verso; Lindgren, "Astronomische und geodätische Instrumente."

[91] Zinner, *Deutsche und niederländische astronomische Instrumente,* 207–10; Fritz Schmidt, *Geschichte der geodätischen Instrumente und Verfahren im Altertum und Mittelalter* (1935; reprinted Stuttgart: Konrad Wittwer, 1988), 328; and Kiely, *Surveying Instruments,* 194–206.

[92] Apian, *Cosmographia,* fol. XVv–XVIv.

[93] Zinner, *Deutsche und niederländische astronomische Instrumente,* 223–25.

[94] Zinner, *Deutsche und niederländische astronomische Instrumente,* 187–91.

[95] Apian, *Instrument Buch,* Cii verso.

[96] Wunderlich, *Kursächsische Feldmeßkunst,* 67–79.

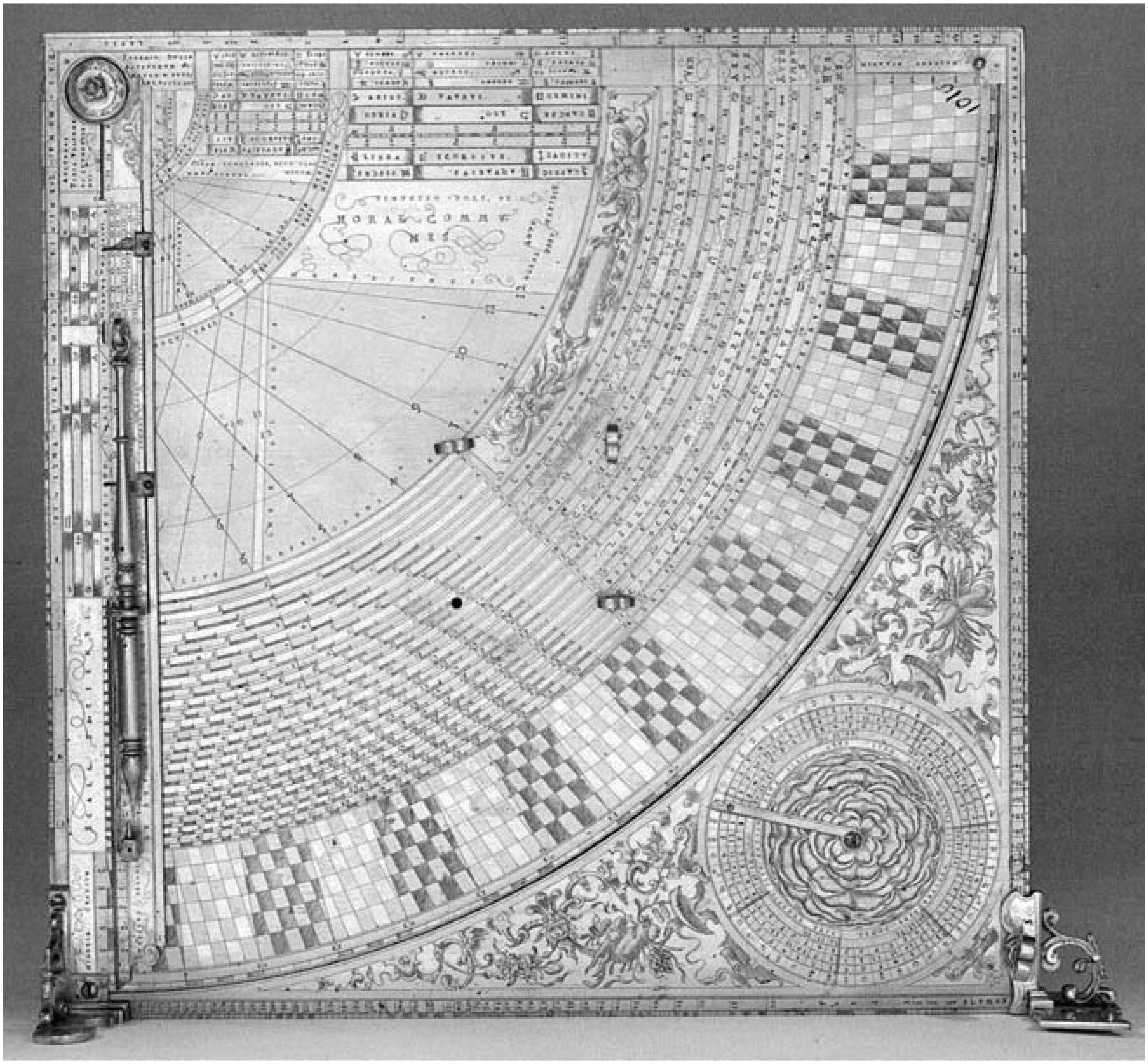

FIG. 19.12. GOLD-PLATED QUADRAT AS A UNIVERSAL INSTRUMENT BY TOBIAS VOLCKMER, 1608. The quadrat (shadow square) incorporates a quadrant with several scales, and in the spare corner it has on one side a magnetic compass and on the other a nonius. This is a complex instrument that not only reveals the magnetic and the real north, the equal and the unequal (welsh) hours of the sun, and the moon's hour, but even allows the calculation of the daily horoscope. Size of the original: 36 × 36 cm. Photograph by Franca Principe, courtesy of the IMSS (inv. no. 2465, 1495).

in 1608 (fig. 19.12).[97] Because of its weight, it was equipped with a tripod support. An attempt was made to convert the quadrant to a universal instrument by the addition of other, sometimes removable, instruments.

During the Renaissance the *triquetum* (also known as a *Dreistab*) was widely used in surveying, although it was less appreciated in astronomy and navigation.[98] Its tradition goes back to classical times. In the early modern period, it was regularly modified by adding full circles to the pivot point for the calculation of angles and a magnetic compass on the side for obtaining the north-south bearing. All three arms, which were pivoted at two points,

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[97] The Medici Stanza delle Matematiche in Florence obtained the instrument. Miniati, *Museo di storia,* 27 and 29, pl. 27.

[98] Schmidt, *Geschichte der geodätischen Instrumente,* 193, 369–81, pl. XXIV, figs. 4 and 7. Kiely devoted some research to this instrument, but supposed a special triangulation function. The study of the treatises on practical geometry shows that none of the instruments was strictly tied to one mathematical method; see Kiely, *Surveying Instruments,* 220–24.

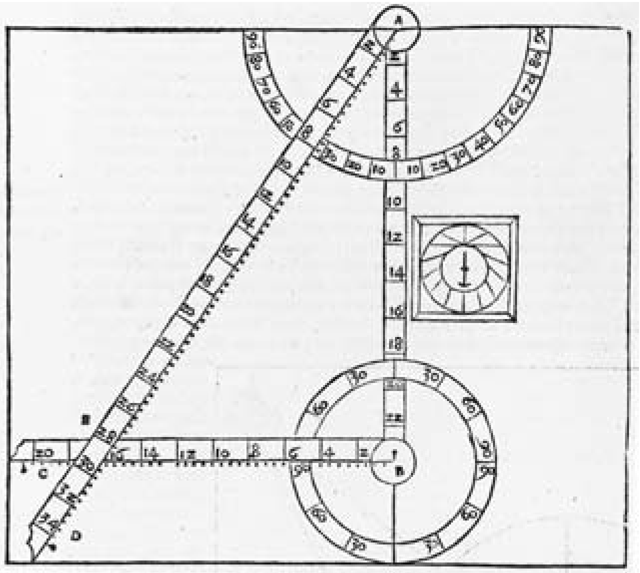

FIG. 19.13. *DREISTAB* WITH A PROTRACTOR AND MAGNETIC NEEDLE COMPASS FROM MÜNSTER, 1550. Design of a *Dreistab* (instrument made of three staffs) with a rather rough scale. The upper pivot is a protractor with a ten-degree interval, and the lower one has a thirty-degree interval; the magnetic compass alongside the middle staff served to determine the north-south direction from which angles were taken.
Size of the original: ca. 12 × 13.9 cm. Sebastian Münster, *Cosmographiae uniuersalis* (Basel: Apud Henrichum Petri, 1550), 24. Photograph courtesy of the BL (566.i.14).

were divided linearly. The device was especially versatile in the Euclidean-based triangulation seen in the work of Sebastian Münster, Daniel Schwenter, and others (figs. 19.13 and 19.14).

The special characteristic of Schickard's *triquetum* was the joining together of the three arms to form an equilateral triangle.[99] At the ends of the staffs Schickard fixed alidades, and he placed movable alidades along the sides. He wanted to divide the necessary scale with values found in the table of tangents. However, he forgot to mention that he needed to draw the height of a triangle as a guide line, because only from this was he able to determine the tangent value. Then he was able to continue the line to the side of his triangle. Whether this brought distinct advantages in practice, where only the most accurate angle calculation would lead to good cartographic results, has not been proven, because this device did not find wide use. Schickard claimed that his *triquetum* was lighter than an equally large circular disk.

Another tradition, also traced back to classical times, used two pivoting arms and was described by Münster and Leonhard Zubler.[100] A precisely constructed instrument with two staffs (*Zweistab*) by Lucas Brunn and Christoph Trechsler the Elder from 1609 was owned by the Dresden Kunstkammer.[101] This instrument was equipped with a finely divided scale and a micrometer slide for exact settings.

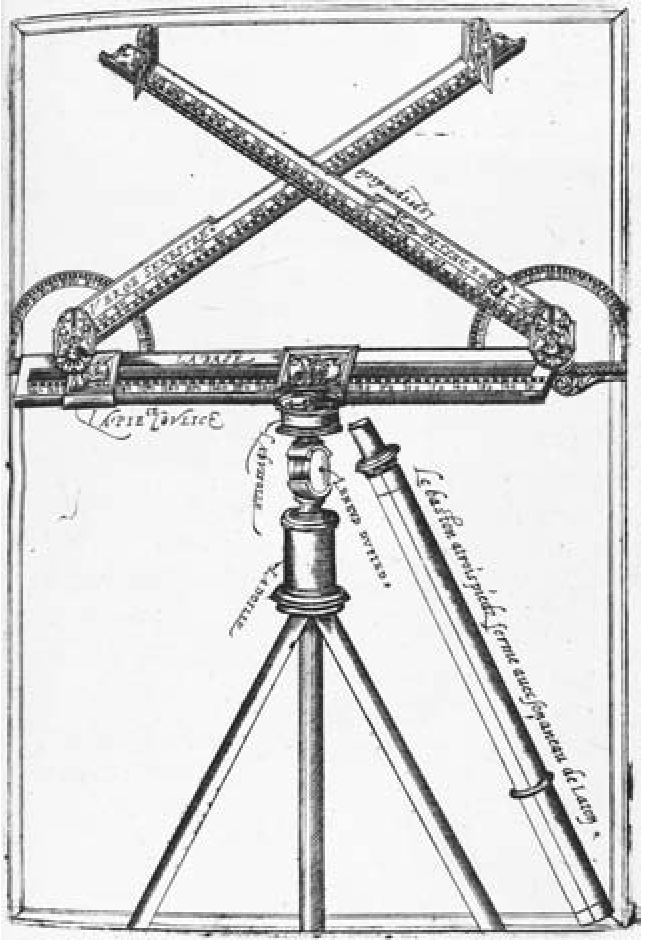

FIG. 19.14. DRAWING OF A *DREISTAB* BY DANFRIE, 1597. This *Dreistab* has precisely carved scales on the staffs and the protractors. One of the staffs, with its protractor, is even movable on a rail; both staffs are armed with alidades.
Size of the original: ca. 15.2 × 10.2 cm. Philippe Danfrie, *Déclaration de l'vsage du graphometre* (Paris, 1597), pt. 2, p. 11. Photograph courtesy of the BL (531.g.7).

A further instrument with classical roots, which in modern times went under the name theodolite, had evidently fascinated the instrumentmakers since the fifteenth century without actually being used practically

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[99] Schickard, *Kurtze Anweisung,* 14.

[100] Wunderlich, *Kursächsische Feldmeßkunst,* 140, and Arthur Dürst, *Philipp Eberhard (1563–1627) & Leonhard Zubler (1563–1611): Zwei Zürcher Instrumentenmacher im Dienste der Artillerie (Ein Beitrag zum Zürcher Vermessungswesen des frühen 17. Jahrhunderts)* (Zurich: Kommissionsverlag Beer, 1983).

[101] Wunderlich, *Kursächsische Feldmeßkunst,* 131–32, 135.

FIG. 19.15. ENGLISH THEODOLITE, 1590. Augustine Ryther's theodolite can be turned vertically (here the semicircle functions as a protractor) and horizontally (the circle also functions as a protractor) and has a magnetic compass at its center. The top balance is fitted with alidades, and shadow squares are incorporated into the circles.
Size of the original: 23.5 cm diameter. Photograph by Franca Principe, courtesy of the IMSS (inv. 240).

(fig. 19.15).[102] This device could be turned in the horizontal plane as well as in the vertical by means of cogged wheels and was described for the first time by Heron of Alexandria (ca. 100 B.C.) under the name *dioptra.* In the fifteenth century it became known as the *torquetum* due to its rotating ability,[103] from which it came to be known as the *Türkengerät* (Turkish device) in colloquial German. Martin Waldseemüller named it the *polimetrum* because of its versatility.[104] It first became a widely used precision instrument in the eighteenth century, when a telescope was mounted instead of the alidade.

Innovations (*Instrumenta Nova*)

Although it is difficult to identify a clear point at which there was a turn from traditional to modern instruments, the inventiveness and willingness to develop ideas in the instrument market during the Renaissance were astounding, even though many alterations hailed as innovations changed neither the construction nor the application of traditional instruments. Consider, for example, the increased size of the sector of the circle. Philipp Apian designed an instrument, the *Triens,* whose scale was larger than that of a quadrant.[105] Thomas Geminus invented a combination quadrant that one could double in size and, by means of a wooden connecting piece, could extend to over 180 degrees.[106] The tendency, however, was to attempt to reduce the whole circle to a useful minimum (the angle and, therefore, the scale): the sextant down to 60 degrees and the octant to 45 degrees. These tools, which from the last quarter of the sixteenth century (an early example comes from Jost Bürgi in Cassel)[107] had been successful in navigation,[108] were also used in land measurement.

The ring instruments represented a fundamental innovation, because their smaller dimensions made them useful as traveling instruments (fig. 19.16). Gemma's *Usus annuli astronomici* marks the beginning of this development, which was successful well into the eighteenth century.[109] They served as sun clocks as well as surveying instruments, but they were usually limited in their accuracy.

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[102] Zinner, *Deutsche und niederländische astronomische Instrumente,* 191–92. The term "theodolit" probably comes from Leonard Digges, who was the first Englishman to explain an instrument similar to that of Heron, Regiomontanus, and Waldseemüller, with the difference that his instrument lacked gears and could not be moved into all positions, including slanting. See Taylor, *Mathematical Practitioners,* 167; Kiely, *Surveying Instruments,* 180–84; and Richeson, *English Land Measuring,* 61–64.

[103] Johannes Regiomontanus et al., *Scripta clarissimi mathematici M. Ioannis Regiomontani, de Torqueto . . .* (Nuremberg, 1544; reprinted Frankfurt am Main: Minerva, 1976), and Lindgren, "Astronomische und geodätische Instrumente," 49.

[104] Waldseemüller finished the illustration for Gregor Reisch in 1515. See Lindgren, "Astronomische und geodätische Instrumente," 61.

[105] See Zinner, *Deutsche und niederländische astronomische Instrumente,* 163–64, under the instruments for the measurement of time.

[106] Conserved now in the Museo di Storia della Scienza, Florence; see Miniati, *Museo di storia,* 32 and 33, pl. 55, and Turner, *Elizabethan Instrument Makers,* 12–23.

[107] Zinner, *Deutsche und niederländische astronomische Instrumente,* 268–76.

[108] Miniati, *Museo di storia,* 14 and 15, pl. 75.

[109] Gemma's *Usus annuli astronomici* first appeared in *Petri Apiani Cosmographia, per Gemmam Phrysium* (Antwerp, 1539). The invention was attributed to Johannes Stabius; see Zinner, *Deutsche und niederländische astronomische Instrumente,* 539. According to Taylor, the ring instrument was also a favorite of some English surveyors and

FIG. 19.16. ASTRONOMICAL RING INSTRUMENT FROM GEMMA. The astronomical ring consists of three metallic rings that bear a scale of degrees, from one degree to ninety degrees, for a quarter of the circle. The scale allows the observation of the stars and of the moon by night and also allows all sorts of land measurements. The shadows of the two thorns on one of the inner rings allow the observer to determine the height of the sun without damaging the eyes. A loop of rope allows the observer to hang the ring at his latitude (fifty-five degrees, shown in the example, is the latitude of Gemma's university town, Louvain).
Size of the detail: ca. 10.1 × 10.1 cm. *Petri Apiani Cosmographia, per Gemmam Phrysium* (Antwerp, 1540), fol. LIIII. Photograph courtesy of the BL (531.g.10[2]).

FIG. 19.17. EXAMPLE OF A SIGHTING TUBE ON A MULTIFUNCTIONAL INSTRUMENT, 1557. On top of this multifunctional instrument, its maker, Baldassarre Lanci, fixed a sighting tube for better sighting with the additional help of a sharp pointer below. The round table that bears the sighting device is engraved with images of field surveying.
Height of the tripod: 139 cm; diameter of table: 30 cm. Photograph by Franca Principe, courtesy of the IMSS (inv. no. 152, 3165).

Innovations can also be seen in the area of the development of sighting tubes to replace the alidade. Considering the wealth of ideas that were applied to instruments, it is surprising that sighting was not recognized as a problem much earlier. Although sighting tubes were used on instruments for mining surveying in the sixteenth century, it was not until 1555 that they were described for use in land surveying. This was Abel Foullon's *holomètre,* a tablelike device with a sighting tube (figs. 19.17 and 19.18)[110] that inspired Philippe Danfrie to construct his *graphomètre* in 1597.[111] Other types of sighting aids were target disks and signal fires, which at the beginning of the seventeenth century were used in Switzerland to make the needed points more easily recognizable in the distance.[112] Surveyors found plenty of targets to aim at in Germany's landscape, including church steeples and other towers.

In an illustration by Jean de Merliers from 1575 showing the process of measuring with the aid of a chain, one can see in the foreground a sighting instrument that made it easier to take bearings of a distant point (fig. 19.19).[113] A wooden quadrant stands horizontally at the eye level of the surveyor. Four straight grooves, forty-five degrees

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scholars, including William Buckley, Ottuel Holynshed, and John Dee; see Taylor, *Mathematical Practitioners,* 314–15 and 318.

[110] Abel Foullon, *Descrittione, e uso dell'holometro* (Paris, 1555; Venice, 1564 and 1584), and *La corte il mare i mercanti,* 146.

[111] Philippe Danfrie, *Déclaration de l'vsage du graphometre* (Paris, 1597); *La corte il mare i mercanti,* 146; Turner, *Early Scientific Instruments,* 253; and Koenraad van Cleempoel, *A Catalogue Raisonné of Scientific Instruments from the Louvain School, 1530–1600* (Turnhout: Brepols, 2002), 205.

[112] Otto Stochdorph, "Abraham (v.) Höltzl (1577/78–1651): Ein Tübinger Kartograph aus Oberösterreich (Bericht)," in *4. Kartographiehistorisches Colloquium Karlsruhe 1988,* ed. Wolfgang Scharfe, Heinz Musall, and Joachim Neumann (Berlin: Dietrich Reimer, 1990), 221–23, esp. 223, and Arthur Dürst, "Der Zürcher Kartograph Hans Conrad Gyger (1599–1674) und sein Werk," in *6. Kartographiehistorisches Colloquium Berlin 1992,* ed. Wolfgang Scharfe (Berlin: Dietrich Reimer, 1994), 139–51, esp. 145.

[113] Lindgren, "Astronomische und geodätische Instrumente," 53.

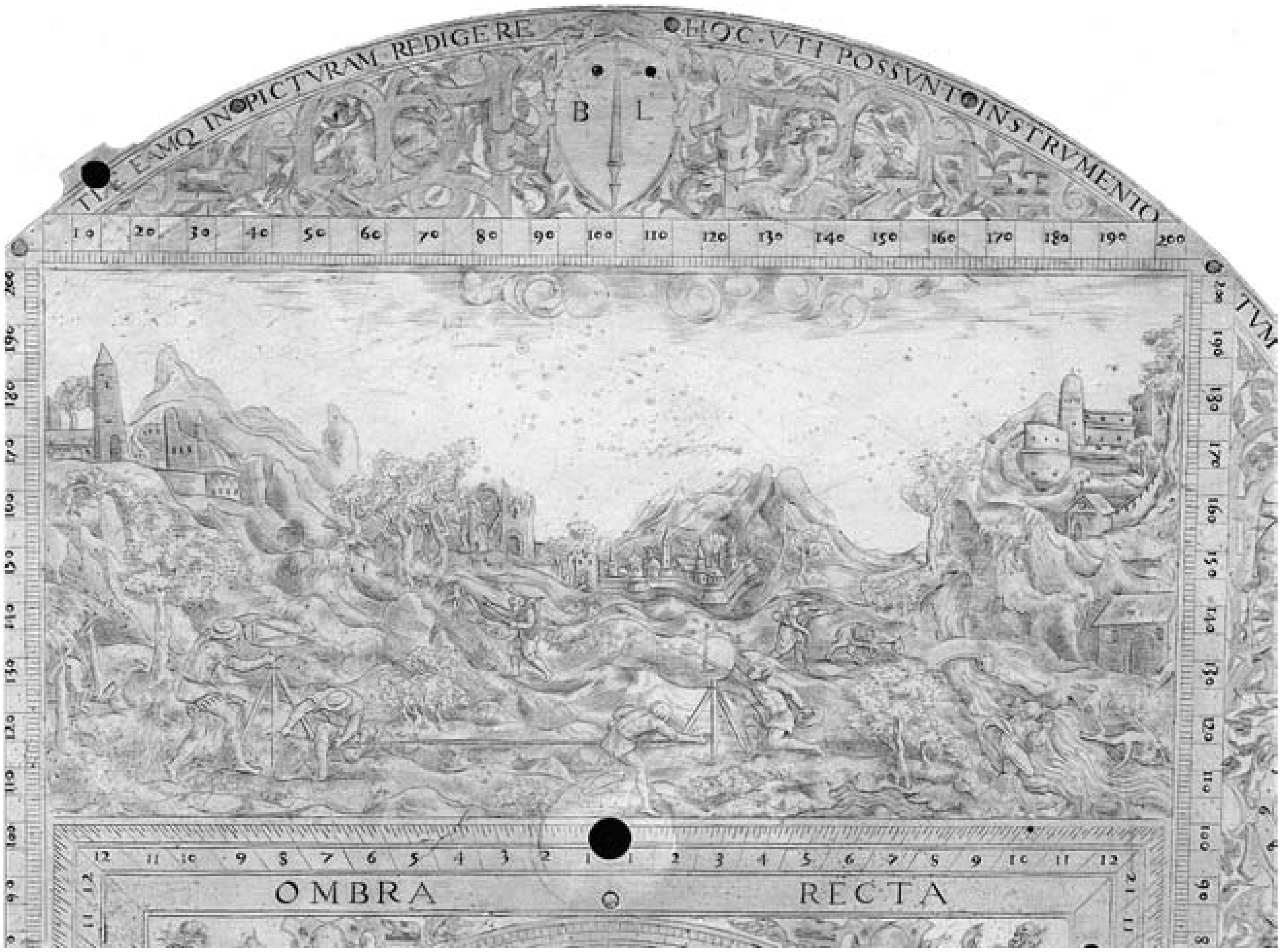

FIG. 19.18. DETAIL OF LANCI'S INSTRUMENT (FIG. 19.17). Radius from center hole: 15 cm. Photograph by Franca Principe, courtesy of the IMSS (inv. no. 152, 3165).

apart, have been cut into the wood. Inscribed inside the quadrant is a circle, at the middle point of which the four grooves meet (why there must be four such grooves instead of only one or two is not clear). This instrument guided the observer's line of sight in a similar way as did the sighting tube with which Reinhold had equipped his mountain quadrant instead of using the normal "sighting hole" (alidade).[114] Such sighting aids are missing from most of the instruments.

The Magnetic Compass

Sebastian Münster used the magnetic compass at the beginning of his surveying discourses due to its circular form as a measuring tool for angles, but later he believed the larger semicircular or circular disks were better suited to this task. Even so, he incorporated a magnetic compass into all of the instruments he used. This served to identify the north-south direction from which all other angles could then be measured. This method can lead to problems due to magnetic declination. For bearings that lie close to each other, this does not play a role, because the angles are always based on the same direction line, even if this is not exactly north-south. But when this method is used, as recommended with Gemma and Schwenter's plane table, over a greater region without any checking taking place—and we must assume that this happened—the change in declination can have consequences. Nothing about this is mentioned in writings on surveying, even though variations in declination had been known from the fourteenth century.[115]

Disregarding such variations for a moment, in the sixteenth century there was speculation that one could ulti-

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[114] Wunderlich, *Kursächsische Feldmeßkunst,* 28.

[115] Battista Agnese, *Portulan-Atlas München, Universitätsbibliothek, cim 18, Farbmikrofiche-Edition,* with "Untersuchungen zu Problemen der mittelalterlichen Seekartographie und Beschreibung der Portulankarten" by Uta Lindren (Munich: Ed. Lengenfelder, 1993), 12 and 14–15, and Klemm, *"Von der Krafft und Tugent des Magneten."*

FIG. 19.19. MEASURING WITH A SIGHTING INSTRUMENT AND CHAIN, 1575. The sighting device (right) with straight grooves carved into the board substituted for the alidade. The use of chains hung on rods required very strong assistants, because the weight of the chain caused a sag that prevented accurate measurement of the straight-line distance.
Size of the original: ca. 15.1 × 15.2 cm. Jean de Merliers, *La practique de geometrie descripte et demostree . . .* (Paris, 1575), fol. A iij. Photograph courtesy of the BL (529.i.30).

mately find a connection between magnetic declination and geographical longitude that would make the determination of longitude easier. This speculation can first be read in Giovanni Battista Della Porta's book *De magnete,* although Della Porta did not claim to be the originator of this idea.[116] Della Porta, a humanist scholar, founded an academy in Naples after extended travels. *De magnete* was part of his chief work, *Magia naturalis,* which was first published as a small booklet in 1557, then as a longer version 1589. Most of *Magia naturalis* has alchemistic and magical content, but the long chapter on magnetism is an interesting statement of contemporary knowledge enriched with some popular jokes. The idea about magnetic declination and longitude was amplified by Athanasius Kircher, who evidently also taught this in Rome.[117] Dutch mariners, especially the sailors of North Atlantic waters, had recorded magnetic declinations in tables quite early. For example, one can find these aids printed by Adriaan Metius, a physician and mathematician who studied in Franeker, Leiden, and Denmark with Tycho Brahe and later became a mathematics professor at the University of Franeker in 1598, but also by Kircher.[118]

Sebastian Cabot wrote a commentary about the magnetic declination in the North Atlantic to accompany a depiction of America.[119] On some Portuguese world maps of the sixteenth century, a slanting bar is shown that indicates that the magnetic needle in the North Atlantic no longer points approximately north, but northwest.[120]

The Plane Table

The surveyor's plane table allowed angles of bearings or directions of bearings to be added directly onto paper. Daniel Schwenter was the first to describe this as a geometrical instrument; he called it the *geometrisches Tischlein* or *mensula Praetoriana,* because he thought it had been invented by his teacher and predecessor at the University of Altdorf, the mathematician Johannes Prätorius, about 1590 (fig. 19.20).[121] It was a portable device consisting of a wooden quadrat with a robust frame mounted on a tripod. Set in the quadrat was a magnetic compass, and on one of the frame sides a movable ruler had been mounted on a rail. In addition, a geometrical drawing device was included, but this was of course not fixed in position. The surface area of the quadrat was covered with paper on which the bearings were drawn.

Without naming Prätorius as inventor and without giving the device a name, Paul Pfinzing, a Nuremberg patrician, described the measuring table as early as 1589 and

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[116] Della Porta, *Magiæ natvralis;* see also Uta Lindgren, "*De Magnete,*" *Morgen-Glantz* 13 (2003): 137–47.

[117] Kircher, *Magnes,* 461–506; Osvaldo Baldacci, "The Cartographic Validity and Success of Martino Martini's Atlas Sinensis," in *Martino Martini geografo, cartografo, storico, teologo: Atti del Convegno Internationale,* ed. Giorgio Melis (Trent: Museo Tridentino di Scienza Naturali, 1983), 73–88, esp. 84–85; and Lindgren, "*Martinis Novus Atlas* Sinensis," 128.

[118] Adriaan Metius, *Geometria practica* (Franeker, 1625); Uta Lindgren, "Adriaan Metius: Nieuwe Geographische Onderwysinghe," in *Copernicus,* 277–78; and Kircher, *Magnes,* 446–52. Metius's publications on astronomy, geography, and nautical themes were widespread and were translated into several languages. I should also mention William Gilbert, who wrote *De Magnete magneticisque corporibus et de magno magnete telluve Physiologia nova* (London, 1600). He owed much to Della Porta and influenced Kircher. See Heinz Balmer, *Beiträge zur Geschichte der Erkenntnis des Erdmagnetismus* (Aarau: H. R. Sauerländer, 1956), 149–63.

[119] Sebastian Cabot, *Declaratio chartae novae navigatoriae domini Almirantis* ([Antwerp], 1544), and Uta Lindgren, "Trial and Error in the Mapping of America during the Early Modern Period," in *America: Early Maps of the New World,* ed. Hans Wolff (Munich: Prestel, 1992), 145–60, esp. 152 n. 15.

[120] Heinrich Winter, "The Pseudo-Labrador and the Oblique Meridian," *Imago Mundi* 2 (1937): 61–73, and E. G. R. Taylor, "Hudson's Strait and the Oblique Meridian," *Imago Mundi* 3 (1939): 48–52.

[121] Schwenter, *Mensula Praetoriana;* see also Georg Drescher, "Wolfgang Philipp Kilian: Johannes Praetorius," in *Copernicus,* 142–43, and Menso Folkerts, "Johannes Praetorius (1537–1616)—Ein bedeutender Mathematiker und Astronom des 16. Jahrhunderts," in *History of Mathematics: States of the Art,* ed. Joseph W. Dauben et al. (San Diego: Academic Press, 1996), 149–69.

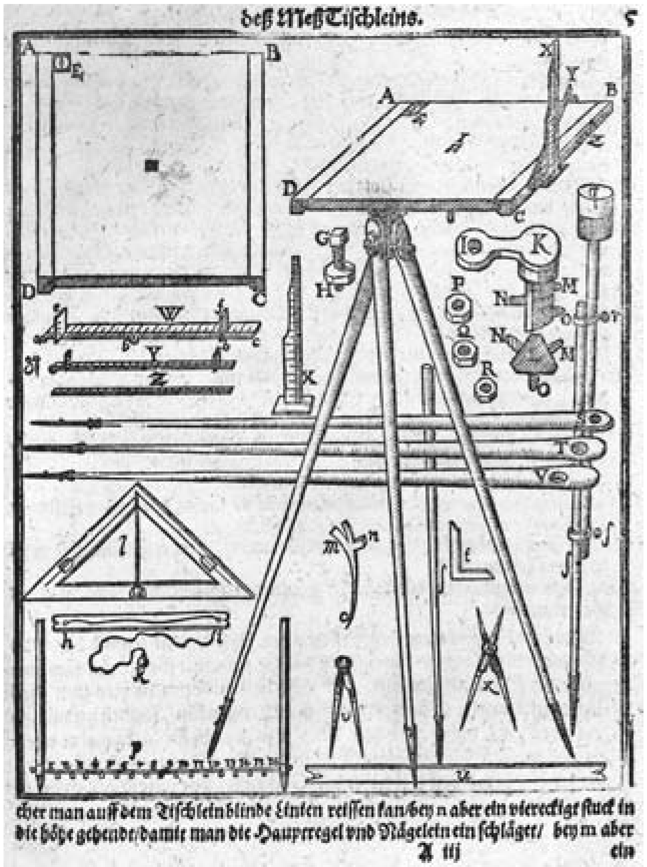

FIG. 19.20. COMPONENTS OF THE SURVEYOR'S PLANE TABLE. The plane table needs a tripod that is depicted here mounted among its components. The table itself consists of a framed board with a magnetic compass in one corner and a movable visor on the frame. This visor can move along the side of the square and also up and down. The frame includes a linear scale. All further geometrical designing devices move freely.
Size of the original: 14.5 × 11.6 cm. Daniel Schwenter, *Geometria practica nova* (Nuremberg, 1641), bk. 3, p. 5. By permission of Houghton Library, Harvard University (Shelfmark GC6 Sch 982 B641g).

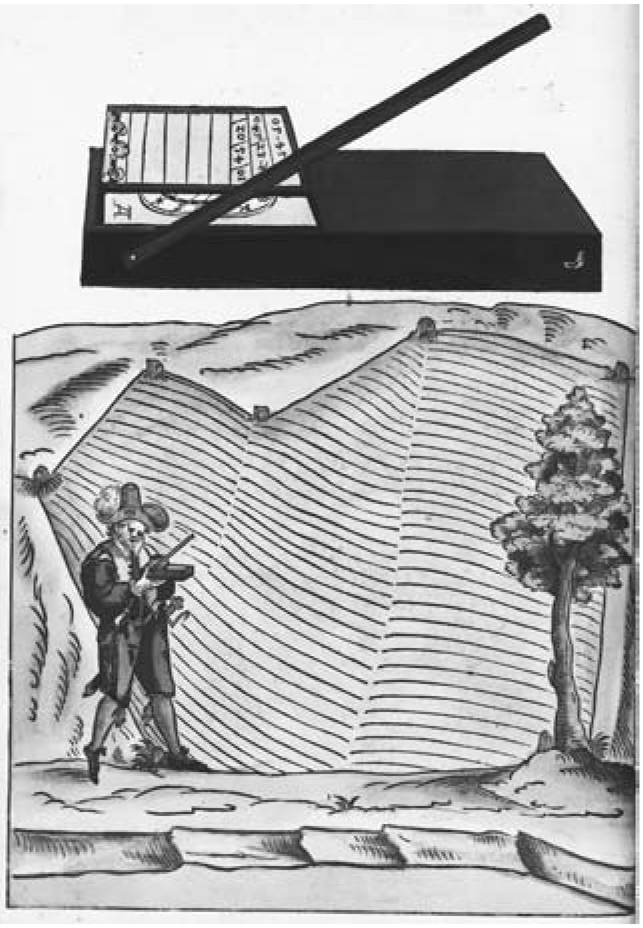

FIG. 19.21. PREDECESSOR OF THE SURVEYOR'S PLANE TABLE, 1598. The surveyor holds an oblong wooden table with a magnetic compass, a simple visor that moves up and down, and ample space to take notes.
Size of the original: 24.3 × 17.8 cm. Paul Pfinzing, *Methodus Geometrica* (Nuremberg, 1598). By permission of Houghton Library, Harvard University (fGC5.P4806.598m).

praised its application.[122] Often more informative than Pfinzing's words, however, are his illustrations (figs. 19.21 and 19.22). One wonders about his verbal inaccuracies, which possibly arose from a lack of rhetorical skills but certainly not from a lack of know-how. The work was reprinted, unaltered, in 1598. There is another description of the plane table from the same time by Cyprian Lucar in *A Treatise Named Lucar Solace* (1590).[123]

In 1607 the Swiss engineer Leonhard Zubler published a tract on the construction and application of the plane table, which he named *Instrumentum Chorographicum.*[124] He ascribed the invention of the plane table to his compatriot mathematician Philipp Eberhard.

The beginnings of the plane table seem to go further back, to the middle of the sixteenth century. No one claimed to be the table's inventor, but it seemed to have had several fathers. Gemma made use of a similar device without noting that this was a new and very practical instrument. Maybe he got the idea from Alberti's drawing device, which he used in his own ways. Likewise, Leonard Digges used the back of his "topographical instrument" to plot out the observed bearings as if it were a plane

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[122] Paul Pfinzing, *Methodus Geometrica, Das ist: Kurtzer wolgegründter unnd außführlicher Tractat von der Feldtrechnung und Messung* (1589; reprinted Neustadt an der Aisch: Verl. für Kunstreprod. Schmidt, 1994), XXVIIff. Pfinzing speaks of an "entry at the Richtscheidt."

[123] Richeson, *English Land Measuring,* 77–81; Taylor, *Mathematical Practitioners,* 328 and 330; and Kiely, *Surveying Instruments,* 230–34. Kiely did not know the German tradition, but assumed that the practical use of the plane table was infrequent in England. On the Continent, the instrument was a great success for many centuries.

[124] Dürst, *Philipp Eberhard,* 22–24.

FIG. 19.22. USING PFINZING'S TABLE, 1598. The surveyor explains his plane table to an older man sitting comfortably as an observer. Here the table has a visor that is as movable as Schwenter's (fig. 19.20). The magnetic compass and a scale are mounted at the visor.
Size of the original: 22.3 × 15.3 cm. Paul Pfinzing, *Methodus Geometrica* (Nuremberg, 1598). By permission of Houghton Library, Harvard University (fGC5.P4806.598m).

table.[125] As early as 1552, Augustin Hirschvogel described a device that must have been very similar to the plane table: he spoke of six different quadrants that he used to record his city plan of Vienna.[126] The six quadrants actually consisted of six sheets of paper, which were mounted (one after the other) on a fixed disk with a magnetic needle, a scale with an alidade, and a length of string.[127] Especially interesting is the fact that the sighting device is upright, which reminds one of Pfinzing's first device, which he rather loosely called a compass.[128]

How Surveyors or Mapmakers Obtained Their Knowledge

This section introduces the scientific backgrounds of some key authors who wrote on practical geometry and surveying, although some of them were also active in theoretical debates. Knowledge about the ability to determine geographic coordinates astronomically remained at the level of late medieval astronomy; it could be gained at most universities where the Quadrivium was taught. The fundamentals of astronomy being taught came out of Johannes de Sacrobosco's *Sphaera mundi,* which itself went no further than book two of the *Natural History* of Pliny the Elder.[129] This explains the fixation, for example, on the antiquated and inappropriate method for determining longitude using the eclipses of the moon. This is curious, because in other fields of study authors usually prided themselves on knowledge of the latest discoveries, which would have been the lunar distance method for this problem.

Johannes Stöffler wrote his short tract on practical geometry more than thirty years after he matriculated at the newly founded University of Ingolstadt in 1472.[130] Stöffler studied for three years, and a short time later took over as parish priest in his hometown, Justingen (near Blaubeuren), where he remained for over thirty years.[131] When Stöffler wrote his tract on geometry in 1511, he had just become—on the urging of Duke Ulrich of Württemberg—professor of mathematics at Tübingen. In Stöffler, Sebastian Münster found at Tübingen a teacher who taught practical geometry and, more particularly, land surveying.[132] In those days, students had to do their bachelor's degree in the faculty of arts before being allowed to change to another faculty. Arithmetic, geometry, and astronomy (along with music) were part of the basic

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[125] Kiely, *Surveying Instruments,* 230–31.

[126] Augustin Hirschvogel's instructions do not have a title; they exist in several manuscripts, for example, Vienna, Österreichische Nationalbibliothek, Cod. 10.690.

[127] Fischer, "Hirschvogels Stadtplan von Wien," 8. Hirschvogel reportedly used six quadrants, and this made a difference for the devices used by Alberti, Gemma, Pfinzing, Schwenter, and those who advocated using the plane table.

[128] Pfinzing, *Methodus Geometrica,* fig. preceding XIX.

[129] Lindgren, "Johannes de Sacrobosco," 221, and Eberhard Knobloch, "Johannes de Sacrobosco . . . Sphaera," in *Copernicus,* 224–25.

[130] Zinner, *Deutsche und niederländische astronomische Instrumente,* 543–45, and Ruthardt Oehme, *Die Geschichte der Kartographie des deutschen Südwestens* (Constance: Jan Thorbecke, 1961), 139–41.

[131] Christoph Schöner, *Mathematik und Astronomie an der Universität Ingolstadt im 15. und 16. Jahrhundert* (Berlin: Duncker und Humblot, 1994), 191–94, esp. 193 n. 20.

[132] Karl Heinz Burmeister, *Sebastian Münster: Eine Bibliographie mit 22 Abbildungen* (Wiesbaden: Guido Pressler, 1964), 10.

curriculum of the Quadrivium. Exactly how thoroughly the mathematical-astronomical studies were covered depended largely on the teacher. But none of those who began teaching geometry at Ingolstadt are known by name.

Stöffler's *De geometricis mensurationibus rerum* covers only a very small part of the *Elements* of Euclid, but it deals with practical examples that a land surveyor, architect, or engineer might need. Specifically, the introduction of trigonometry had evidently been completed by the time Stöffler studied at Ingolstadt. Later he liked to look back on his period of study at Ingolstadt, but he had not shown any particular interest in mathematics in those days. The tutors for the faculty of arts were masters recruited from Vienna, where in the previous decades a famous mathematical-astronomical school had developed to which Georg von Peuerbach and Johannes Regiomontanus belonged.[133]

Jakob Köbel was a humanistically educated publisher and politician in Oppenheim on the Rhine.[134] He had achieved a bachelor's degree in law at Heidelberg, and in 1490 studied mathematics and astronomy at the University of Cracow. In 1492 he returned to the Palatinate and printed a number of Stöffler's works. His practical geometry is not his own work; he merely published Stöffler's work in German.

Peter Apian, from Leisnig in Saxony, was forty-three years younger than Stöffler.[135] He had studied from 1516 to 1519 at the University of Leipzig and the following two years in Vienna, where in 1521 he was awarded his bachelor's degree. Leipzig was the first choice for students from Saxony (including Regiomontanus), but Vienna was always attractive. Apian's mentors have not been identified, although his mathematical work is linked to the Viennese School and especially to Regiomontanus's writings on trigonometry.[136] From 1526 until the end of his life, he occupied the chair of mathematics and astronomy at the University of Ingolstadt, where he transferred and continued the printing office that he had started at Landshut.

In 1513 Johann Scheubel made his way from Kirchheim unter Teck to study in Vienna.[137] Twenty years later, he enrolled at Leipzig and obtained his bachelor's degree. In 1535 he matriculated at Tübingen, was awarded a master's degree in 1540, proceeded to teach courses on geometry, and was made *Euclidis profesor ordinarius* in 1550. A sketch of the boundaries of the city of Esslingen (1556–57) and a map of Württemberg (1558-59) are ascribed to him, neither of which was based on astronomical-geometrical principles.

Oronce Fine was one year older than Apian.[138] He learned his first lessons in mathematics from his father, a doctor in Briançon. After his father's early death, he was accepted at the Collège de Navarre in Paris. Due to his specialized interest in mathematics and astronomy, he soon left and matriculated at the University of Paris, where he obtained a master's degree in the faculty of arts. From 1518 until 1524, he was in prison, probably for a failed horoscope. From 1525 until his death, he occupied the chair of mathematics in Paris. Although his surveying instructions are similar to Stöffler's, it is not clear whether Stöffler's work was part of the curriculum at the Paris University or whether Fine came across Stöffler through his own reading.

Reiner Gemma Frisius went to school in Groningen, studied at the University of Louvain under unknown teachers, and gained his doctorate in medicine.[139] He then taught at Louvain as a professor in the medical faculty. He instructed Gerardus Mercator in the construction of astronomical instruments and globes as early as the 1530s[140] and taught the English cosmographer John Dee.[141]

Interest in mathematical-astronomical questions must have been unusually great in the Low Countries toward the beginning of the sixteenth century. Gemma was only sixteen years old when Apian's *Cosmographicus liber* (1524) first appeared in Landshut. The fact that in the same year it was republished in Antwerp shows how well it was received there. From 1529 Apian's work appeared with Gemma's commentaries and supplementary writings. From the

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[133] Schöner, *Mathematik und Astronomie,* 192; Vogel, "Das Donaugebiet"; Günther Hamann, "Regiomontanus in Wien," and Paul Uiblein, "Die Wiener Universität, ihre Magister und Studenten zur Zeit Regiomontans," both in *Regiomontanus-Studien,* ed. Günther Hamann (Vienna: Verlag der Österreichischen Akademie der Wissenschaften, 1980), 53–74 and 395–432; and Helmuth Grössing, ed., *Der die Sterne liebte: Georg von Peuerbach und seine Zeit* (Vienna: Eramus, 2002).

[134] Benzing, *Jakob Köbel.*

[135] Zinner, *Deutsche und niederländische astronomische Instrumente,* 233–34, and Hans Wolff, "Im Spannungsfeld von Tradition und Fortschritt, Renaissance, Reformation, und Gegenreformation," in *Philipp Apian,* 9–18, esp. 16–17.

[136] Menso Folkerts, "Die Trigonometrie bei Apian," in *Peter Apian: Astronomie, Kosmographie und Mathematik am Beginn der Neuzeit,* ed. Karl Röttel (Buxheim: Polygon, 1995), 223–28.

[137] Ulrich Reich, "Johann Scheubel (1494–1570), Wegbereiter der Algebra in Europa," in *Rechenmeister und Cossisten der frühen Neuzeit,* ed. Rainer Gebhardt (Freiberg: Technische Universität Bergakademie Freiberg, 1996), 173–90.

[138] Adolphe Rochas, *Biographie du Dauphiné* (1856; reprinted Geneva: Slatkine Reprints, 1971); Leo Bagrow, *Meister der Kartographie,* rev. ed., ed. R. A. Skelton (Berlin: Safari, 1963), 487; and Knobloch, "Oronce Finé," 188.

[139] Zinner, *Deutsche und niederländische astronomische Instrumente,* 320–21; Marcel Watelet, "De Rupelmonde à Louvain," in *Gérard Mercator cosmographe: Le temps et l'espace,* ed. Marcel Watelet (Antwerp: Fonds Mercator Paribas, 1994), 72–91, esp. 75–79; and Van Cleempoel, *Catalogue Raisonné,* 9–11.

[140] Watelet, "De Rupelmonde à Louvain," 76–79, and Elly Dekker and Peter van der Krogt, "Les globes," in *Gérard Mercator cosmographe: Le temps et l'espace,* ed. Marcel Watelet (Antwerp: Fonds Mercator Paribas, 1994), 242–67, esp. 243.

[141] Bagrow, *Meister der Kartographie,* 186, and Tyacke and Huddy, *Christopher Saxton,* 21.

second half of the sixteenth century, interest in nautical methods and aids was more widespread in the Low Countries, which in the areas of mathematics and astronomy was often based on the same principles as cartography. Gemma's interest was not in delving into the mathematical foundations; his contribution inspired methods and instruments for the practice of terrestrial surveying.

Another Dutch university that emphasized scientific research and teaching was that at Leiden. Willebrord Snellius was there as successor to his father, Rudolf, in the chair of mathematics.[142] Adriaan Metius studied in Leiden and in Franeker before he, too, became professor of mathematics in Franeker in 1598.[143] His manuals, which were used in Altdorf among other places, were still being translated and distributed in the eighteenth century.[144] In addition to mathematics, the curriculum consisted of land surveying, navigation, military engineering, and astronomy.

Philipp Apian continued the Ingolstadt tradition.[145] Initially he was taught by his father and a private teacher. In 1537 the duke's ten-year-old son Albrecht (later Albrecht V of Bavaria), a little older than Philipp, was sent each day to Peter Apian for instruction in cosmography, geography, and mathematics. At the age of eighteen, Philipp was sent on a study trip that led him to Strasbourg, Dôle, Paris, and Bourges, from which he returned shortly before the death of his father in 1552. He was elected successor to his father's teaching chair, and in 1554 was given the task of mapping Bavaria. By 1561 the surveying, which had occupied the summer months, had been accomplished (Apian taught his courses in the winter months).[146]

Philipp Apian had to vacate his chairs twice for religious reasons. In 1569 he not only had to leave Ingolstadt, but had to leave Bavaria completely because he had refused to swear the Tridentine profession of faith. That same year he became professor of astronomy and geometry at Tübingen University. When he refused to sign the Formula of Concord in 1583, he was forced to leave the university but was allowed to remain in Tübingen, where he died in 1589.

Apian was, after Stöffler and Scheubel, the third mathematician at the university in Tübingen who took the improvement of maps to heart. Apian's successor was Michael Mästlin, a professor in Tübingen from 1580, who had been a student of Apian.[147] He, in turn, was the teacher of Johannes Kepler and Wilhelm Schickard. Another pupil of Mästlin's during the years 1597–1602 was the Austrian Abraham von Höltzl, who produced a map of the Schwäbischen Kreis about 1620.[148]

Schickard had studied in Tübingen and was appointed Mästlin's successor as professor of mathematics in 1631, having been professor of Hebrew since 1619.[149] He was the fifth Tübingen mathematics professor to have cartographic interests. In 1624 he had begun a systematic record of the land of Württemberg because he was dissatisfied with the maps available. After his early death from the plague, the complete and final drawings of thirteen map sheets are said to have been sent to Amsterdam for printing. Only sheet eight, showing Tübingen and the surrounding area, is still in existence.[150]

Erasmus Reinhold (d. 1574) was the little-known son of the Wittenberg astronomer of the same name, who died at the early age of forty-two in 1553.[151] He came from Saalfeld in Thüringia,[152] where his son also set up business as a doctor. It is probable that he studied under his father in Wittenberg. From Reinhold we have the tract mentioned earlier as well as maps of the districts of Altenburg and Eisenberg in Thüringia.

The cartographer Tilemann Stella from Siegen also studied in Wittenberg before going to Marburg for two years. He returned to Wittenberg and Cologne between 1546 and 1551, before he was summoned to Schwerin to the court of the duke of Mecklenburg as a mathematician and geographer with the special responsibility to determine boundaries. From 1582 until his death in 1589, he lived at the court of the Count Palatine bei Rhein in Zweibrücken. In 1560 he carried out angle measurements from the tower of St. Stephan's Cathedral in Vienna to the most prominent features of the city. He confided this only in his diary. His maps have been executed as oil paintings on canvas; they have been neither copied nor thoroughly studied concerning their accuracy.[153]

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[142] Christianson, *Tycho's Island,* 358–61.

[143] Knobloch, "Praktisches Geometrie," 137–38, and Uta Lindgren, "Adriaan Metius: Institutiones Astronomicae & Geographicae," in *Copernicus,* 252.

[144] Herbert J. Nickel, *Joseph Sàenz de Escobar und sein Traktat über praktische und mechanische Geometrie: Eine Anleitung zur angewandten Geometrie in Neuspanien (Mexiko) um 1700* (Bayreuth: Universität Bayreuth, Fachgruppe Geowissenschaften, 1998), 27 n. 49.

[145] Stetter, "Apian," 70.

[146] Rüdiger Finsterwalder, *Zur Entwicklung der bayerischen Kartographie von ihren Anfängen bis zum Beginn der amtlichen Landesaufnahme* (Munich: Verlag der Bayerischen Akademie der Wissenschaften in Kommission bei der C. H. Beck'schen Verlagsbuchhandlung, 1967), 20–23.

[147] Richard A. Jarrell, "Astronomy at the University of Tübingen: The Work of Michael Mästlin," in *Wissenschaftsgeschichte um Wilhelm Schickard,* ed. Friedrich Seck (Tübingen: J. C. B. Mohr [Paul Siebeck], 1981), 9–19.

[148] Stochdorph, "Abraham (v.) Höltzl," 222–23.

[149] Zinner, *Deutsche und niederländische astronomische Instrumente,* 500–501.

[150] Werner Stams, "Die Anfänge der neuzeitlichen Kartographie in Mitteleuropa," in *Kursächsische Kartographie,* 37–105, esp. 88.

[151] Wunderlich, *Kursächsische Feldmeßkunst,* 19–20.

[152] Fritz Bönisch, "Kleinmaßstäbige Karten des sächsisch-thüringischen Raumes," in *Kursächsische Kartographie,* 1:207–47, esp. 245.

[153] Christa Cordshagen, "Tilemann Stella—Ein Leben für die Kartographie," in *9. Kartographiehistorisches Colloquium Rostock 1998,*

Daniel Schwenter studied mathematics from 1602 at Altdorf University, which was a part of Nuremberg territory, under Johannes Prätorius.[154] Already as a pupil at school in Sulzbach, he had shown a special interest in geometry and had studied, among other subjects, Hirschvogel's geometry from 1543. In 1608 he became professor of Hebrew studies at Altdorf, and in 1628 he obtained the chair of mathematics. The later Regensburg accounting teacher Georg Wendler, who had studied in Altdorf under Schwenter's successor Abdias Trew (from 1636 professor of mathematics), wrote of practical terrain exercises in which he and Trew made sightings of Altdorf from a nearby hill and carried out measurements.[155]

Augustin Hirschvogel came from Nuremberg and served an apprenticeship as glass painter under his father, Veit.[156] Nothing is known about his school education, but it seems clear that he must have attended at least one of the many Nuremberg *Rechenschulen* on the evidence of his later achievements.[157] In 1543 he published a *Geometria,* paying particular attention to the ideas of perspective.[158] In 1547, after doing other cartographic work, he produced a city plan of Vienna based on measurements. In 1552 he was writing about his geometrical methods and the various instruments used[159] and finally earned the honorary title of "Mathematicus" in Vienna.[160] He must have become familiar with instrumentmakers and their products while in Nuremberg; the instructions contain one of the earliest known descriptions of a plane table. Because Hirschvogel never claimed to be the inventor of this instrument, it is assumed that he took knowledge gained in Nuremberg with him to Vienna.

Paul Pfinzing, too, came from Nuremberg, where he was a merchant.[161] He began to study in Leipzig in 1562 at the age of eight—the University at Altdorf was not opened until 1575—and in 1594 presented his hometown with the atlas that he had produced—as well as other maps—himself.[162] The accuracy of Pfinzing's maps has not yet been studied. He is the only cartographer of the period under study who carried out cartography more or less as a hobby.

Although born and raised in Windsheim, Sebastian Kurz later went to Nuremberg to become a teacher. He attended only the *Rechenschule* in Windsheim and later became principal of this school for a short time.[163] In 1617 he published a *Tractatus geometricus,* having already translated a book on the practice of surveying (*Practica des Landvermessens*) as well as descriptions of instruments from Dutch in 1616. The author of the *Practica* (original title, *Practijck des lantmetens*) was surveyor Jan Pietersz. Dou of Leiden.[164]

The vast majority of the authors on surveying during the sixteenth and seventeenth centuries had university studies behind them and had become university professors. However, as one can see from the last two examples, this was not the only educational path of such authors. Abraham Ries, the most talented among the children of the *Rechenmeister* (computation teacher), and mathematician Adam Ries from Staffelstein, had enrolled at the

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ed. Wolfgang Scharfe (Bonn: Kirschbaum, 2002), 13–20. On Stella's climb up the tower of St. Stephan's cathedral, see Fischer, "Augustin Hirschvogels Stadtplan von Wien," 8. On Tilemann Stella (Stolz), see also Leo Bagrow, "A. Ortelii catalogus cartographorum," *Petermanns Geographische Mitteilungen,* Ergänzungsheft 199 (1928): 1–137, with plates, and 210 (1930): 1–135, esp. 70–77; Peter H. Meurer, *Fontes cartographici Orteliani: Das "Theatrum orbis terrarum" von Abraham Ortelius und seine Kartenquellen* (Weinheim: VCH Acta Humaniora, 1991), 244–47; Gyula Pápay, "Aufnahmemethodik und Kartierungsgenauigkeit der ersten Karte Mecklenburgs von Tilemann Stella (1525–1589) aus dem Jahre 1552 und sein Plan zur Kartierung der deutschen Länder," *Petermanns Geographische Mitteilungen* 132 (1988): 209–16, esp. 209; idem, "Ein berühmter Kartograph des 16. Jahrhunderts in Mecklenburg: Leben und Werk Tilemann Stellas (1525–1589)," in *Beiträge zur Kulturgeschichte Mecklenburgs aus Wissenschaft und Technik* (Rostock: Wilhelm-Pieck-Universität Rostock, Sektion Geschichte, 1985), 17–24, esp. 19; Stams, "Anfänge der neuzeitlichen Kartographie," 83–84 nn. 308 and 309; Bönisch, "Kleinmaßstäbige Karten," 237–41; and the essay collection *Tilemann Stella und die wissenschaftliche Erforschung Mecklenburgs in der Geschichte* (Rostock: Wilhelm-Pieck-Universität Rostock, 1990).

[154] Folkerts, "Johannes Praetorius," 159.

[155] Menso Folkerts, "Georg Wendler (1619–1688)," in *Rechenbücher,* 335–45, esp. 152.

[156] Fischer, "Hirschvogels Stadtplan von Wien"; idem, "Stadtpläne und Veduten Wiens"; and Andreas Kühne, "Augustin Hirschvogel und sein Beitrag zur praktischen Mathematik," in *Verfasser und Herausgeber mathematischer Texte der frühen Neuzeit,* ed. Rainer Gebhardt (Annaburg-Buchholz: Adam-Ries-Bund, 2002), 237–51.

[157] Adolf Jaeger, "Stellung und Tätigkeit der Schreib- und Rechenmeister (Modisten) in Nürnberg im ausgehenden Mittelalter und zur Zeit der Renaissance" (Ph.D. diss., Friedrich-Alexander Universität Erlangen-Nürnberg, 1925).

[158] Augustin Hirschvogel, *Ein aigentliche und grundtliche anweysing in die Geometria* (Nürnberg, 1543); Fischer, "Hirschvogels Stadtplan von Wien," 3; and Kühne, "Augustin Hirschvogel," 239.

[159] Fischer, "Hirschvogels Stadtplan von Wien," 8, and note 126 in this chapter.

[160] Kühne, "Augustin Hirschvogel," 240.

[161] Ernst Gagel, *Pfinzing: Der Kartograph der Reichsstadt Nürnberg (1554–1599)* (Hersbruck: Im Selbstverlag der Altnürnberger Landschaft, 1957), 2, and Peter Fleischmann, introduction to *Das Pflegamt Hersbruck: Eine Karte des Paul Pfinzing mit Grenzbeschreibung von 1596,* by Paul Pfinzing (Nuremberg: Altnürnberger Landschaft e. V. in collaboration with the Staatsarchiv Nürnberg, 1996).

[162] Gagel, *Pfinzing,* 4. Pfinzing's 1594 atlas (71 cm × 51 cm) is a bundle of handwritten and designed maps with no official title. See Peter Fleischmann, *Der Pfinzing-Atlas von 1594: Eine Ausstellung des Staatsarchivs Nürnberg anlässlich des 400 jährigen Jubiläums der Entstehung,* exhibition catalog (Munich: Selbstverlag der Generaldirektion der Staatlichen Archive Bayerns, [1994]), and Hans Wolff, ed., *Cartographia Bavariae: Bayern im Bild der Karte,* exhibition catalog (Weißenhorn, Bavaria: A. H. Konrad, 1988), 60.

[163] Kurt Hawlitschek, "Sebastian Kurz (1576–1659): Rechenmeister und Visitator der deutschen Schulen in Nürnberg," in *Rechenbücher,* 257–66, esp. 257 and 259.

[164] Hawlitschek, "Sebastian Kurz," 265.

University of Leipzig, but probably never studied there.[165] Abraham was instructed by his father, and after the latter's death in 1559 he took over the management of the *Rechenschule* in Annaberg in the Erzgebirge as well as the office of recorder (of the owners of mining shares) for the elector. In 1559 he created a map of the Obererzgebirgischen Kreis based on measurements he had carried out himself and in 1575 created another of the Vogtländischen Kreis. Both areas were newly added territories of Saxony. The primary objective of Ries in carrying out the measurements was to determine the size of the new districts. A gold-plated multipurpose instrument with the basic form of an astrolabe, produced by Abraham Ries, was kept in the elector's Kunstkammer and could still be found there in 1874.

At the same school was Lucas Brunn from Annaberg.[166] He later studied in Leipzig and Altdorf (under Prätorius) and obtained his master's degree. He constructed precision instruments and was very possibly the inventor of the adjustable-screw micrometer. In 1619 he became "Inspektor" of the Dresden Kunstkammer, and in 1625 he published *Euclidis Elementa practica.*

A complete exception to the university background of authors of surveying manuals was Nicolaus Reimers of the illustrious lineage "de Baren" (Ursus) of Dithmarschen.[167] He first learned to read and write at the age of eighteen, but due to his evident talent he was chosen to serve Heinrich von Rantzau, the Danish governor of southern Schleswig-Holstein in 1573/74. One of Rantzau's jobs was to survey the property, a task carried out for tax purposes and for which it was important to calculate areas very carefully. Whether the surveying in fact led to the construction of a map is not known, but there must have been this intention at least, because in 1583 Reimers had a surveying manual printed to which he gave the title *Geodæsia Rantzoviana,* which described the duties of the ancient *agrimensores* and the measuring of field area.[168]

Although various triangulation and astronomical methods were the primary means used to survey land, there were also the methods of the ancient *agrimensores,* but this second means of surveying the land was seldom seen in Germany until the end of the sixteenth century. However, it is evident in the book *Fundamentum geographicum,* a teaching manual on the construction of maps that Caspar Dauthendey published in 1639, shortly before his death.[169] He taught geometry and geography and drew and published a map of Brunswick (Braunschweig) on the basis of his own mathematical observations. In his work he complained of the difficulties of land surveying, which was more important than ever after the destruction caused during the Thirty Years War. He especially noted that the field surveyor was unqualified due to a lack of proper geometrical knowledge, while the somewhat higher-ranked geometers shrank from carrying out the work of measuring. He therefore suggested that the field surveyors be provided with better geometrical education.[170] Here we have two completely separate terms being used: field surveyor and geometer.

The German empire of the Middle Ages was constituted as a *Personenverbandsstaat* (state as an association of persons), in which privileges and rights were not connected with the possession of land but rather awarded to individuals. Therefore, field surveying in the ancient tradition made little sense.

In England, the profession of "surveyor" had been known for a longer time, and its legal interest came from large land distributions after the dissolution of the monasteries. The surveyor's task was the administration of large estates and their supervision, for which an overview of the extent of the land was necessary. For this reason, one finds pictorial representations that—depending on quality, scale, and format—can be regarded as maps.[171] Christopher Saxton and John Norden were primarily "surveyors" who were also busy producing maps.[172] We know little about Saxton's education. As a young man, from about 1554 to 1570 he was a servant to the Dewsbury clergyman John Rudd, from whom it is likely that he received his training. Rudd himself had made journeys in 1561 in order to obtain information for the construction of maps, but nothing is known about his methods, and Rudd's maps have not survived. It is assumed that Saxton accompanied him.[173] Until about 1587, Saxton was always someone's servant, which makes his lifestyle completely different from those of the German mapmakers. From 1587 he worked for himself as a "surveyor."

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[165] Hans Wußing, *Die Coß von Abraham Ries* (Munich: Institut für Geschichte der Naturwissenschaften, 1999), and Peter Rochhaus, "Adam Ries in Sachsen," in *Adam Rieß vom Staffelstein: Rechenmeister und Cossist* (Staffelstein: Verlag für Staffelsteiner Schriften, 1992), 107–25.

[166] Zinner, *Deutsche und niederländische astronomische Instrumente,* 266, and Wunderlich, *Kursächsische Feldmeßkunst,* 130–35.

[167] Dieter Launert, *Nicolaus Reimers (Raimarus Ursus): Günstling Rantzaus—Brahes Feind* (Munich: Institut für Geschichte der Naturwissenschaften, 1999).

[168] Launert, *Nicolaus Reimers,* 134–45.

[169] Fritz Hellwig, "Caspar Dauthendey und seine Karte von Braunschweig," *Speculum Orbis* 2 (1986): 25–33, esp. 26.

[170] Hellwig, "Caspar Dauthendey," 30.

[171] Tyacke and Huddy, *Christopher Saxton,* 24, and Ifor M. Evans and Heather Lawrence, *Christopher Saxton: Elizabethan Map-Maker* (Wakefield, Eng.: Wakefield Historical Publications and Holland Press, 1979).

[172] Frank Kitchen, "John Norden (c. 1547–1625): Estate Surveyor, Topographer, County Mapmaker and Devotional Writer," *Imago Mundi* 49 (1997): 43–61.

[173] Tyacke and Huddy, *Christopher Saxton,* 24.

Links between Surveying and Maps

We know from only very few contemporary accounts which methods and instruments were used to construct the many maps produced in the period covered by this chapter. In many cases, a quantitative study of accuracy tells us more about a map than all the earlier or modern praise or criticism. However, not all maps are suitable for an analysis of their exactness. Without going into detail as to which scale should be set as an upper limit, it is absolutely clear that for world or continent maps—despite all the obvious differences—the question of their relationship to astronomical-geometrical surveying in the early modern age is problematic. General maps produced during this time, such as those of the larger countries like France, Italy, and the German empire, should not be included in the discussion. None of these countries was measured terrestrially during the early modern era, and the number of locations with reliable coordinates was much too small. Bavaria, which in those days extended only as far north as the Danube and westward to the river Lech, was the largest state on the Continent to be mapped in the sixteenth century on the basis of astronomical and geometrical methods. All other newly produced maps with claims to accuracy were of smaller areas. Only by keeping this background in mind is it worthwhile considering the question of precision. And only then can the question of establishing a link between the previously mentioned surveying techniques and instrumentation and the maps themselves be posed.

Geographical drawings and even paintings are preserved in the larger cartographic collections, for which an analysis of accuracy is neither possible nor indeed worthwhile. They were produced during the early modern period as an *Augenschein* (a kind of eyewitness evidence) of a given landscape.[174] For example, a draftsman who was sworn in by a court would have captured the situation involving a legal dispute over land, so the court could gain an impression of the situation without having to examine the location personally and without having to rely on the biased information of one of the parties involved. Never intended as a plan view, but often with several changing perspectives, the landscape represented by the draftsman's drawing lacks any geometrical basis. Other "maps," too, such as those in the *Cosmography* of Sebastian Münster, dispensed with geometrical fundamentals as well as arrangement in a grid system. One cannot apply the standard of accuracy to these sketches.

The question of accuracy is also problematic in changing landscapes. In Ostfriesland, for example, the inhabited area was, in early modern times, only a very narrow ring around a huge moorland.[175] Beyond was the sea, which in those days had anything but an exact boundary. The maps of the astronomer David Fabricius from 1589 and 1592 were greatly improved on by the mathematician Ubbo Emmius in 1595, but the severe storm tides of 1625 made his results largely invalid.[176] In 1627 Johann Conrad Musculus recorded the immense damage done in his map of the dikes.[177] Despite having experience in surveying, he never achieved anything like the quality obtained by Emmius and Fabricius.

Another reason to avoid assessing a map from the early modern era by quantitative methods lies in the great differences in scale within one sheet. For example, districts or towns and cities that were important to the mapmaker or the client were sometimes drawn to a larger scale than other areas, because there was more information in populated areas that was worth showing than in the surrounding area, where there was little of interest to the mapmakers' contemporaries—neither roads nor destinations. This was the case with the maps of England by Christopher Saxton. These changes in scale, however, did not lead to a general distortion of the area represented in Saxton's atlas of England and Wales. Vector analysis of the coordinates of approximately sixty towns shows that the geographical latitude has been measured very well on the whole. When it comes to longitudes, Cornwall and Wales stretch too far toward the west, but others are amazingly good, even if the variations are in different directions. As Skelton wrote, one would like to know whether this result was based on astronomical observations, because the accuracy cannot be coincidental.[178]

Today not only can one answer in the affirmative, but from this precision it is also possible to name the methods that were applied. Because the lunar eclipse method entails an inaccuracy of some twenty to thirty minutes of

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[174] Hans Vollet, "Der 'Augenschein' in Prozessen des Reichskammergerichts—Beispiele aus Franken," in *5. Kartographiehistorisches Colloquium Oldenburg 1990,* ed. Wolfgang Scharfe and Hans Harms (Berlin: Dietrich Reimer, 1991), 145–63.

[175] Arend W. Lang, *Kleine Kartengeschichte Frieslands zwischen Ems und Jade: Entwicklung der Land- und Seekartographie von ihren Anfängen bis zum Ende des 19. Jahrhunderts* (Norden: Soltau, 1962), 29–33 and 41–46.

[176] Bagrow, *Meister der Kartographie,* 486–87, and Menso Folkerts, "Der Astronom David Fabricius (1564–1617): Leben und Werk," *Berichte zur Wissenschaftsgeschichte* 23 (2000): 127–42.

[177] Albrecht Eckhardt, "Johann Conrad Musculus und sein Deichatlas von 1625/26," in *5. Kartographiehistorisches Colloquium Oldenburg 1990,* ed. Wolfgang Scharfe and Hans Harms (Berlin: Dietrich Reimer, 1991), 31–40, esp. 37–39. Compare the similar problems in Dagmar Unverhau, "Das Danewerk in der *Newen Landesbeschreibung* (1652) von Caspar Danckwerth und Johannes Mejer," in *Das Danewerk in der Kartographiegeschichte Nordeuropas,* ed. Dagmar Unverhau and Kurt Schietzel ([Neumünster]: Karl Wachholtz, 1993), 235–57, esp. 236–49.

[178] Skelton, *Saxton's Survey,* 8–9.

time in establishing the exact start and end of full eclipse, which represents five to eight degrees of longitude at a particular latitude, the spread of error would have to be much larger if Saxton had used this method. And Saxton could not have taken the coordinates from anyone else. In 1574—the same year that Saxton began collecting his information on the ground—William Bourne published a list of coordinates in which only London has a relatively exact value; other cities such as Hereford, Oxford, Cambridge, and Bourne's hometown, Gravesend, vary considerably from Saxton's values.[179] Against this background, it becomes clear why Saxton included a border scale for his wall map, which he divided into ten-minute intervals. As opposed to Apian's use of astronomically determined points and applied trigonometry to fill in a single-scale *Landtafeln,* Saxton chose to depict towns in greater detail than the surrounding areas, with corresponding differences in scale, so it is hard to imagine that Saxton could have carried out an exact terrestrial survey, more or less according to the geometrical methods of Gemma, even if, as in the case of Wales, he was issued a pass by which he was permitted access to towers, castles, higher-lying locations, and hills so he could see the lay of the land.[180] After 1587, Saxton conducted estate surveys; his last known work was completed in 1608.[181]

Saxton's contemporary, the "surveyor" John Norden, left a series of maps, but unfortunately none of his methods. No study of his accuracy has been carried out, as has been done for Saxton's wall map. Norden was different from Saxton inasmuch as the making of maps was for a time his chosen professional aim, but one he had to abandon because payment was not guaranteed.[182] He set store by the ability of the user of his maps to ascertain the distance between two towns from them. But that does not permit one to conclude anything about the methods used.

The military engineer Robert Lythe carried out surveying work in southern Ireland in the years 1567–70 and on the basis of his efforts drew up a map that included no coordinates.[183] One learns little of his methods from his own writings. He traveled mainly by boat on Irish rivers, and he presumably measured latitudes and longitudes; he could not get the necessary overview for terrestrial surveying from a boat.[184] He expressly vowed to work according to the "rules of cosmography." Thus, one is surprised at the lack of coordinates on his map. However, he did carry out measurements approximately every five miles, providing the English administration with a much-improved knowledge of the land hydrography.[185]

One of the methods used by Saxton, Norden, and Lythe in order to learn more about an area in question was to seek the accompaniment and assistance of local guides who could provide them with the names of settlements, river routes, woods, and forests.[186] This must have considerably speeded up such surveying journeys. The maps and sketches of these "surveyors" suggest—as far as is known—a readiness to pay attention to detail, revealing a different purpose behind these representations than is inferred from the more generalized style of regional maps.

A similar difference exists between the plans constructed by engineers for irrigation and drainage in parts of Italy (e.g., Venice) and regional maps.[187] The so-called *periti* who specialized in this kind of work had limited terms of office and clearly defined instructions. They did not need any special geometrical training, as shown by the experience of the artist and engineer Cristoforo Sorte from Verona.[188] From 1556 to 1564 and from 1589 to 1593 he was *perito ordinario,* and in this role he produced a large number of drawings of irrigation and drainage systems. He was working for the Camera ai Confini in the years following 1570 and mapped the alpine border region near Venice. Another *perito,* Giacomo Gastaldi, born in Piedmont, was busy carrying out surveying work on the river Adige between 1550 and 1556.[189] These very large-scale plans did not lead to any direct improvement in the construction of even his own published maps. An example of how unrealistically Gastaldi himself illustrated the river system in the Venetian terra firma is a map published without a title shortly after his death, with Padua as its centerpiece.[190]

The establishment of borders also played a major role for the Grand Duchy of Florence. The Archivio di Stato in Florence has a separate department, the Archivio dei Confini, in which, among other works, there are ten large folio volumes of topographical maps.[191] In a map from 1643, Lunigiana (north of Carrara) is illustrated—totally unrealistically—as if the border is formed by a ring of

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[179] W. L. D. Ravenhill, "'As to Its Position in Respect to the Heavens,'" *Imago Mundi* 28 (1976): 79–93, esp. 82.

[180] Tyacke and Huddy, *Christopher Saxton,* 32.

[181] Tyacke and Huddy, *Christopher Saxton,* 46.

[182] Kitchen, "John Norden," 46–48.

[183] J. H. Andrews, "The Irish Surveys of Robert Lythe," *Imago Mundi* 19 (1965): 22–31.

[184] Andrews, "Irish Surveys," 24.

[185] Andrews, "Irish Surveys," 24.

[186] Tyacke and Huddy, *Christopher Saxton,* 32, and Skelton, *Saxton's Survey,* 9.

[187] Denis E. Cosgrove, "Mapping New Worlds: Culture and Cartography in Sixteenth-Century Venice," *Imago Mundi* 44 (1992): 65–89, esp. 67–75.

[188] Cosgrove, "Mapping New Worlds," 72.

[189] Cosgrove, "Mapping New Worlds," 74.

[190] Published in Venice, 1569. See Valeria Bella and Piero Bella, eds., *Cartografia rara: Antiche carte geografiche, topografiche e storiche dalla collezione Franco Novacco* (Pero, Milan: Cromorama, 1986), 102.

[191] Mario Tesi, ed., *Monumenti di cartografia a Firenze (secc. X–XVII),* exhibition catalog (Florence: Biblioteca Medicea Laurenziana, 1981), 36–41.

mountains around a wide valley basin.[192] This style of representation harks back to the Roman tradition, wherein mountains could be imagined to form borders, which was never adopted in any European mountain area. Detailed topographical knowledge did not, however, necessarily lead to realistic regional maps. The map of Toscana, with a grid in the borders and based on a woodcut by Girolamo Bell'Armato of 1536, which was reprinted by a variety of publishers as a copper engraving until 1646, shows the Tiber and Arno connected by the Chiana, in error.[193] In reality, today's highly fertile Chiana Valley was then, in fact, a dreaded swamp area that was first drained dry in the second half of the eighteenth century by canals on which the famous mathematicians and engineers Evangelista Torricelli and Vincenzo Viviani worked. Although the Chiana Valley belongs topographically to the river system of the Tiber, part of the water volume is now fed through the Canal Maestro into the Arno. Even further removed from reality is the Toscana map by Leonardo da Vinci, although the strikingly modern style is highly convincing. Here the Chiana Valley is represented as a long lake with two bulges, giving the whole the appearance of a bird with its wings outstretched.[194]

While the multitude of small independent states in Italy could pose a hindrance to the recording of information necessary for map production, the countries of Spain and Portugal, which were to be united under the Spanish crown in 1580 for a period of about seventy years, found themselves in the opposite situation.[195] These countries were politically not fragmented, and additionally had acquired, since the second half of the fifteenth century, extremely large coastal areas of Africa, eastern Asia, and America. This meant an immense task for the surveyors and the cartographers. In the Archivo General de Simancas are a large number of harbor plans mirroring the interests of the global powers.[196] Here, too—as in Italy—we have, side by side, plan-style drawings of irrigation projects and systems and moderately successful maps of the provinces.

Irrigation canals were not only planned but also implemented for many locations in arid Spain. Extremely impressive was the planning behind the Acequia Imperial de Aragón in the Ebro Valley.[197] This was started by Charles V and was largely completed under Philip II, with final completion of the canal in the eighteenth century. The existing plans from the sixteenth and seventeenth centuries are individually very different. Philip II, who, like his father Charles V, was very interested in mathematical and astronomical instruments, summoned the Portuguese João Baptista Lavanha (Juan Bautista Labanna) to the newly founded Academia de Matemáticas in 1582.[198] After Philip II's death, Lavanha served under his son Philip III. In 1607 he was given the task of surveying and mapping the kingdom of Aragón. During 1610 and 1611, he was, therefore, in the country. Perhaps this was too short a period; a quick comparison of the river system portrayed with a modern map shows the inadequacy of Lavanha's map. It was printed in Madrid in 1619 and was regarded as one of the finest achievements of the seventeenth century. Philip had already commissioned a map of the whole of Spain. In 1566 the cosmographer and mathematics professor Pedro de Esquivel began the fieldwork. He was followed by Diego de Guevara and finally by the architect of the Escorial, Juan de Herrera.[199] The work was definitely intended for the purposes of internal administration.

What is noteworthy is the way that contemporaries valued the geometrical and astronomical principles of maps. If one looks at the mediocre results (i.e., the surviving maps), one ought to take into account the actual enthusiasm of the surveyor—not the standards or the reports. Here is only one characteristic example: Andreas Bureus was the first surveyor and cartographer to serve in the central office of surveying in Sweden (the Lantmäterikontoret), founded in 1628.[200] There was a gaping discrep-

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[192] "Carta della Lunigiana e degli Stati confinanti, delineata nel 1643," Archivio dei Confini VII, 47, Archivio di Stato, Florence; see also Tesi, *Cartografia a Firenze,* 40, pl. XXXIII, and Uta Lindgren, "Die Grenzen des Alten Reiches auf gedruckten Karten," in *Bilder des Reiches,* ed. Rainer A. Müller (Sigmaringen: Jan Thorbecke, 1997), 31–50, esp. 39 and 41.

[193] The map bears the title *Tusciae elegantioris Italiae partis . . . ;* see Bella and Bella, *Cartografia rara,* 140–41.

[194] Windsor Royal Library, nos. 12277 and 12278r. *La corte il mare i mercanti,* 165 (with incorrect archive information); Carlo Zammattio, "Mechanics of Water and Stone," in *Leonardo the Scientist* (New York: McGraw-Hill, 1980), 10–67, esp. 23; and Leonardo da Vinci, *I manoscritti e i disegni di Leonardo da Vinci . . . : I disegni geografici conservati nel Castello di Windsor* (Rome: Libreria dello Stato, 1941), pls. 12 and 14 and figure 36.5 in this volume.

[195] Gonzalo de Reparaz Ruiz, "The Topographical Maps of Portugal and Spain in the 16th Century," *Imago Mundi* 7 (1950): 75–82.

[196] *Felipe II: Los ingenios y las máquinas,* exhibition catalog ([Madrid]: Sociedad Estatal para la Conmemoración de los Centenarios de Felipe II y Carlos V, 1998), 136–83.

[197] *Felipe II: Los ingenios,* 234–37.

[198] Reparaz Ruiz, "Maps of Portugal and Spain," 82, and José Luis Casado Soto, "João Baptista Lavanha: Descripción del reino de Aragón," in *Felipe II: Un monarca y su época. Las tierras y los hombres del rey,* exhibition catalog ([Madrid]: Sociedad Estatal para la Conmemoración de los Centenarios de Felipe II y Carlos V, 1998), 233.

[199] Most likely this work is preserved in what is called the "Escorial Atlas" in the library at the Escorial. See chapter 39 in this volume, esp. figures 39.12 and 39.13; Carmen Líter and Luisa Martín-Merás, introduction to *Tesoros de la cartografía Española,* exhibition catalog ([Madrid]: Caja Duero Biblioteca Nacional, [2001]), 35–48, esp. 38; Casado Soto, "João Baptista Lavanha," 38; and *Felipe II en la Biblioteca Nacional* (Madrid: Ministerio de Educación y Cultura, Biblioteca Nacional, 1998), 75.

[200] On Bureus, who was given the title General-mathematicus, see Ulla Ehrensvärd, Pellervo Kokkonen, and Juha Nurminen, *Mare Balticum: 2000 Jahre Geschichte der Ostsee* (Helsinki: Verlags-AG.

ancy there, too, between theoretical standards and execution. His map of the province Dalecarlia (which included the city Falun), based on surveys and local knowledge, was produced as a result of a border dispute. From the viewpoint of quality, his map surpasses many of the legal maps of the empire, but it satisfies so few of the modern demands for accuracy that the comparison with reality becomes superfluous.[201]

Conclusion

What general themes concerning the theory and practice of land surveying in the Renaissance can be identified? The most important point to be made is that the practice lagged far behind the theory. Although—as Münster so succinctly put it—"everything you measure must be measured by triangles," indeed few maps in the fifteenth, sixteenth, and early seventeenth centuries were based on triangulation. Euclid's ancient method of measuring the position of an inaccessible point by means of similar triangles had long been employed in classical and medieval civil engineering to solve problems such as the alignment of tunnels or measurement of the width of rivers, and these principles were adopted in several surveying instruments, such as the quadrant and square. But the application of triangulation—in which a point could be fixed with one known side and two known angles—although a simple concept, does not seem to have been articulated until Alberti and Regiomontanus did so in the fifteenth century, and it was not provided full explanation in a surveying context until Gemma's treatise of 1533. In the sixteenth century, despite a large number of manuals seeking to explain the principles of triangulation, the graphic solution through the use of the plane table or one of its precursors—such as Gemma's overlapping plotted circles—seems to have been favored over the trigonometrical method using sine and tangent functions. This is not surprising, because use of the plane table was a remarkably elegant and intuitive way of producing a reduced scale model of the landscape directly on paper.

In addition to the lack of practical application of triangulation, the lack of use of astronomically defined coordinates in map compilation is also striking. Although many of the mapmaking principles known in the Renaissance were based on astronomical applications, such as the method of fixing star positions with coordinates, the imprecision of longitude measurements in a terrestrial context rendered this method impractical. Thus, Münster's request for information from administrative civil servants and fellow scholars with which to compile the maps for his *Cosmography* did not generate any information derived from astronomical observations of latitude and longitude. The lack of geodetic control in large-scale maps resulted in a lack of correspondence between manuscript surveyed maps and regional maps intended for publication. Thus, the positional information found on Giacomo Gastaldi's large-scale maps associated with hydrographical management failed to find its way onto the smaller-scale regional maps that bear his name as compiler.

The bulk of the theoreticians were scholars and academics with a background in mathematics, beginning with the influential Johannes Stöffler, whose treatise on practical geometry was modified by several authors in the early sixteenth century. Later in the century they were joined by more amateur mathematical practitioners in England or by the *Rechenmeister* (computation teachers) in Germany. The demand for the services of these scholars and practitioners varied across Europe. Many of them taught practical geometry in the context of the general university course of the Quadrivium, while others, such as Philipp Apian, conducted surveying work primarily under princely patronage. Differences in the legal rights associated with land ownership also affected the demand for land surveying.

Just as the extant maps from the period—with very few exceptions (the maps of Philipp Apian and Christopher Saxton, for example)—cannot be used as primary sources to indicate the use of systematic surveys and triangulation, surviving surveying instruments are not reliable guides to the methods that might have been employed or the precision with which they might have been carried out. Many instruments were designed to demonstrate the ingenuity of their makers rather than to be of immediate utility, and thus they were often far too complicated for a surveyor to understand. Thus, despite early precursors of the theodolite, such as the *polimetrum* or *torquetum,* this instrument did not find wide use until the eighteenth century.

The overall lack of correspondence between theory and practice in land surveying mirrors a similar lag in the general cartography of the Renaissance, where modern methods of compiling maps had been postulated long before observations of sufficient precision were possible. It was not until the eighteenth century that observational practice was to catch up with the mathematical theory.

Otava, 1996), 119 and 198, and Ulla Ehrensvärd, *Sjökortet Gav Kursen* ([Stockholm: Kungl. Bibl.], 1976), 7. On the Swedish land survey, see Fritz Curschmann, "Die schwedischen Matrikelkarten von Vorpommern und ihre wissenschaftliche Auswertung," *Imago Mundi* 1 (1935): 52–57; Nils Friberg, "A Province-map of Dalecarlia by Andreas Bureus(?)," *Imago Mundi* 15 (1960): 73–83; and chapter 60 in this volume.

[201] Friberg, "Province-map of Dalecarlia," 73–74. See also Fritz Bönisch, "The Geometrical Accuracy of 16th and 17th Century Topographical Surveys," *Imago Mundi* 21 (1967): 62–69.